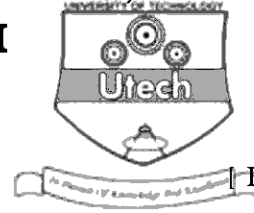


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ENGINEERING & MANAGEMENT EXAMINATIONS, JUNE – 2009

MATHEMATICS – II
SEMESTER – 2



Time : 3 Hours]

Full Marks : 70

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10

i) If $y = 3x$, then $\frac{d^2y}{dx^2}$ is

a) 0

b) 1

c) 2

d) 3.

ii) The value of $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$ is

a) e

b) $1/e$

c) 0

d) 1.

iii) The value of $\int \frac{dx}{x^2 - a^2}$ is

a) $\frac{1}{2a} \log \frac{a+x}{a-x}$

b) $\sin^{-1} \frac{x}{a}$

c) $\log \sqrt{x+x^2+a^2}$

d) $\frac{1}{2a} \log \frac{x-a}{x+a}$

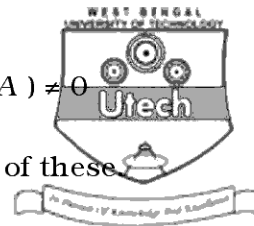
iv) The co-factor of 'c' in the determinant $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$ is

a) $(-1)^{3+3} \begin{vmatrix} a & h \\ h & b \end{vmatrix}$

b) $(-1)^{3+2} \begin{vmatrix} a & h \\ h & b \end{vmatrix}$

c) $(-1)^{2+3} \begin{vmatrix} a & h \\ h & b \end{vmatrix}$

d) none of these.



v) The matrix A is said to be orthogonal matrix if

a) $\det (A) = 0$

b) $\det (A) \neq 0$

c) $A^T A = I$

d) none of these.

vi) A function $f (x)$ is said to be an odd function of x if $f (- x)$ is equal to

a) $-f (x)$

b) $f (x)$

c) $f (- x)$

d) 1.

vii) The value of the determinant $\begin{vmatrix} 3 & 4 & 5 \\ (\sqrt{3})^2 & 2^2 & (\sqrt{5})^2 \\ 4 & 5 & 6 \end{vmatrix}$ is equal to

a) 1

b) 2

c) 23

d) 0.

viii) $\frac{d}{dx}(10^x)$ is equal to

a) $10^x \log_{10} e$

b) 10

c) 10^x

d) $x.10^{x-1}$.

ix) A matrix A is said to be an idempotent matrix if

a) $A^2 = A$

b) $A^2 = I$

c) $A^2 = 0$

d) none of these.

x) If $y = x \log x$, then $\frac{dy}{dx}$ is equal to

a) $1 + \log x$

b) x

c) $\frac{1}{x}$

d) 1.

xi) The focus of $y^2 = 36x$ is

a) (3, 0)

b) (9, 0)

c) (3, 9)

d) none of these.



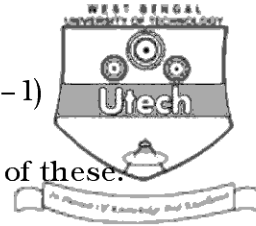
xii) The value of $\int x e^x dx$ is

a) $x e^x$

b) $e^x (x-1)$

c) $x e^x + x$

d) none of these.



xiii) Parametric co-ordinates of the parabola $y^2 = 4ax$ is

a) $(-at^2, -2at)$

b) $(-at^2, 2at)$

c) $(at^2, 2at)$

d) none of these.

xiv) The eccentricity of the ellipse $3x^2 + 4y^2 = 24$ is

a) $\frac{1}{4}$

b) $\frac{3}{4}$

c) $\frac{1}{2}$

d) $\frac{\sqrt{3}}{2}$.

GROUP - B

(Short Answer Type Questions)

Answer any *three* of the following questions.

3 × 5 = 15

2. Solve the following system of equations by the matrix method :

$$x + y + z = 8$$

$$x - y + 2z = 6$$

$$3x + 5y - 7z = 14$$

3. Prove that $\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc$.

4. Evaluate the integral $\int \frac{x-22}{3x^2-2x-8} dx$.

5. Find the value of $A^2 - 4A - 5I$, if $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$.

6. If $f(x) = \sin(\log x)$, then find $f'(x)$.

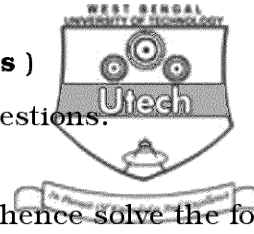


GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following questions.

3 × 15 = 45



7. a) Obtain the inverse of the matrix $\begin{bmatrix} 2 & 4 & -1 \\ 3 & 1 & 2 \\ 1 & 3 & -3 \end{bmatrix}$ and hence solve the following system

of equations :

$$2x + 4y - z = 9$$

$$3x + y + 2z = 7$$

$$x + 3y - 3z = 4$$

- b) Show that the vectors $\alpha_1 = (5, 7, 11)$, $\alpha_2 = (2, 1, 3)$ and $\alpha_3 = (3, 6, 8)$ are linearly dependent.

- c) Find the equation of the ellipse which meets the straight line $\frac{x}{7} + \frac{y}{2} = 1$ on the axis of x and the straight line $\frac{x}{3} + \frac{y}{5} = 1$ on the axis of y and whose axes lie along the axes of co-ordinates. Determine the eccentricity and the co-ordinates of the foci of the ellipse.

6 + 2 + 7

8. a) Find the maximum value of $\left(\frac{1}{x}\right)^x$.

- b) Evaluate the integral $\int \frac{x^2 dx}{\sqrt{1+x^3}}$. 8 + 7

9. a) Evaluate $\lim_{n \rightarrow \infty} \frac{1^m + 2^m + 3^m + \dots + n^m}{n^{m+1}}$ ($m > -1$).

- b) If $y = a \sin (mx) + b \cos (mx)$, then prove that $\frac{d^2y}{dx^2} = -m^2 y$. 8 + 7

10. a) Find the equation of the normal to the parabola $y^2 = 3x$ which is perpendicular to $y = 2x + 4$.

- b) If $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 5 & 1 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1} A^{-1}$.

- c) If $x = a \cos 2t$, $y = a \sin 2t$, then find $\frac{d^2y}{dx^2}$. 5 + 6 + 4



11. a) Prove that the matrix $A = \frac{1}{3} \begin{bmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ is an orthogonal matrix and hence

find A^{-1} .

b) Evaluate the integral $\int \frac{e^x}{x} (x \log x + 1) dx$.



8 + 7

12. a) Find the equation of the parabola whose vertex is $(-1, 3)$ and the focus is $(3, -1)$.

b) Taking the major and minor axes as the axes of the co-ordinates, find the equation of the ellipse whose length of latus rectum is $\frac{32}{5}$ units and co-ordinates of one focus are $(3, 0)$.

c) If S and S' are the foci and P be any point on the hyperbola $x^2 - y^2 = a^2$, prove that $SP \cdot S'P = CP^2$, where C is the centre of the hyperbola.

5 + 5 + 5

END