Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/BBA (H)/BIRM/BSCM/SEM-2/BBA-202/2011

2011
MATHEMATICS – II

Time Allotted : 3 Hours | Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The value of $\lambda t_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ is

a) e b) $\frac{1}{e}$

c) 0 d) 1.

ii) The derivative of xe^x is

a) e^x

b) $e^x(x+1)$

c) $e^x(x-1)$

d) none of these.

iii) If $y = 8x^2$, then $\frac{d^2y}{dx^2}$ is

a) 8 b) 0

c) 16 d) none of these.

iv) A function $f(x)$ is said to be an even function if $f(-x)$ is equal to

a) $f(x)$ b) $-f(x)$

c) $-f(-x)$ d) none of these.

v) The value of $\int_0^1 2e^x dx$ is

a) $2e$ b) 2

c) $2(e-1)$ d) none of these.

vi) The co-factor of c is the determinant $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$ is

a) $(-1)^{3+3}\begin{vmatrix} a & h \\ h & b \end{vmatrix}$

b) $(-1)^{3+2}\begin{vmatrix} a & h \\ h & b \end{vmatrix}$

c) $(-1)^{1+3}\begin{vmatrix} a & h \\ h & b \end{vmatrix}$

d) none of these.

vii) The determinant of an orthogonal matrix is

a) 0 b) 1

c) ± 1 d) none of these.

viii) The focus of $y^2 = 36x$ is

a) (3, 0) b) (9, 0)

c) (3, 9) d) none of these.

ix) Rank of the matrix $\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$ is

a) 1 b) 0

c) 2 d) none of these.

x) The integration of $\int 2^x dx$ is

a) $\frac{2^x}{\log_e 2}$ b) $x\, 2^{x-1}$

c) $2^x \log_e 2$ d) none of these.

xi) A matrix is said to be orthogonal if

a) $\det A = 0$

b) $\det A \neq 0$

c) $A^T A = I$

d) none of these.

xii) The curve $y = |x|$ is

a) differentiable everywhere

b) discontinuous at $x = 0$

c) not derivable at $x = 0$

d) none of these.

xiii) The function ($5x + 3$) is increasing in the interval

a) (0, 5) b) (–1, 5)

c) ($-\infty, -\infty$) d) none of these.

xiv) The function $u(x, y) = \dfrac{(x+y)^2}{(x-y)^2}$ is a homogeneous function of degree

a) 0 b) 1

c) 2 d) none of these.

GROUP – B
(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Verify Euler's theorem for the function $f(x, y) = x^2 + 10xy + y^2$.

3. Show that the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ satisfies the equation $A^2 - 2A - 5I = 0$ and hence find A^{-1}.

4. Find the rank of the matrix $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 4 & 2 \end{pmatrix}$.

5. Find the equation of the parabola whose vertex is (– 2, 2) and focus is (– 6, 6).

6. Solve the following system of equation by Cramer's rule :

$x + y + z = 8$

$x - y + 2z = 6$

$3x + 5y - 7z = 14$.

7. Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

8. a) If $x + y + z = 0$, then show that $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = 0$.

b) Show that the matrix $A = \begin{pmatrix} 2 & -3 & 1 \\ 3 & 1 & 3 \\ -5 & 2 & -4 \end{pmatrix}$ satisfies the equation $A\,(A - I)\,(A + 2I) = 0$.

c) Compute the inverse of the matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$.

9. a) Verify whether the following matrix $A = \frac{1}{3}\begin{pmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ is orthogonal or not. Find A^{-1}.

b) Solve the following system of equation by matrix inversion method :

$x + y + z = 6$

$x - y + z = 2$

$2x + y - z = 1$

c) Find the value of t for which the matrix $\begin{pmatrix} 2 & 0 & 1 \\ 5 & t & 3 \\ 0 & 3 & 1 \end{pmatrix}$ is singular.

10. a) Verify whether the function $f(x)$ as defined below is continuous or not at $x = 2$.

$$f(x) = \begin{cases} x^2 + 4, & x > 2 \\ 8, & x = 2 \\ 3x^2 - 4 & x < 2 \end{cases}$$

b) Find $\frac{d^2y}{dx^2}$ if $x = \frac{t^2}{1+t}$ $y = \frac{t}{1+t}$.

c) If $y = \sin\left(m \sin^{-1} x\right)$ then show that

$$\left(1 - x^2\right)y_2 - xy_1 + m^2 y = 0.$$

11. a) If $y = a \sin(mx) + b \cos(mx)$ then show that $\frac{d^2y}{dx^2} = m^2 y$.

b) If $A = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 0 \\ 5 & 1 \end{pmatrix}$ then verify that $(AB)^{-1} = B^{-1}A^{-1}$.

c) Prove that $\sqrt{3}\sin x + 3\cos x$ has a maximum at $x = \frac{\pi}{6}$.

12. a) If $u = x^2 + y^2 + z^2$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 2u$.

b) Find the equation of the ellipse which meets the straight line $\frac{x}{7} + \frac{y}{2} = 1$ on the x-axis and the straight line $\frac{x}{3} + \frac{y}{5} = 1$ on the y-axis and whose axes lie along the axes of coordinates. Determine the foci of the ellipse.

c) Evaluate $\int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)dx$.

13. a) Find the maximum and minimum value of the function $f(x) = x^3 + \frac{1}{x^3}$.

b) Prove that $\begin{vmatrix} 1 & b+c & b^2+c^2 \\ 1 & c+a & c^2+a^2 \\ 1 & a+b & a^2+b^2 \end{vmatrix} = (a-b)(b-c)(c-a)$.

c) Find the area above the X-axis bounded by $x - 2y + 4 = 0$, $x = 1$ and $x = 9$.