Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/BBA(H)BIRM/BSCM/SEM-2/BBA-202/2013

2013
MATHEMATICS-II

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) The value of the

   $\lim_{x \to 0} (1+x)^{\frac{1}{x}}$ is equal to

   a) 1 b) e

   c) 0 d) ∞ .

   ii) The curve $y = |x|$ is symmetric about

   a) x – axis b) y – axis

   c) $y = x$ d) none of these.

iii) if $\begin{pmatrix} 7 & x+y \\ 8 & x-y \end{pmatrix} = \begin{pmatrix} 7 & 5 \\ 8 & 3 \end{pmatrix}$, then the values of x and y are

a) $x = 4,\ y = 1$ b) $x = 3,\ y = 2$

c) $x = 1,\ y = 4$ d) $x = 0,\ y = 5.$

iv) A function $f(x)$ is said to be odd if $f(-x)$ is equal to

a) $f(x)$ b) $-f(x)$

c) $-f(-x)$ d) none of these .

v) The rank of the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{pmatrix}$$ is

a) 1 b) 2

c) 3 d) None of these.

vi) The parametric coordinates of the parabola

$y^2 = 4ax$ are

a) $(at^2,\ 2at)$ b) $(-at^2,\ -2at)$

c) $(-at^2,\ 2at)$ d) none of these.

vii) The function $f(x, y) = \dfrac{x^3 + y^3}{x^2 + y^2}$ is homogeneous of degree

a) 1 b) 2

c) 3 d) none of these.

viii) The vectors (1, 2, 3), (2, 0, 1) and (3, 2, 4) are linearly independent over R (field of real numbers). This statement is

a) true b) false.

ix) If M is a square matrix of order 3 , then its transpose is a matrix of order

a) 2×3 b) 2×2

c) 3×3 d) 3×2.

x) The value of $\frac{d}{dx}(a^x)$ is equal to

a) a^x b) $a^x \log_e a$

c) $\log_e a$ d) $x\, a^{x-1}$.

xi) The value of $\int \frac{dx}{a^2 - x^2}$ is

a) $\frac{1}{2a} \log \left| \frac{x-a}{x+a} \right|$ b) $\frac{1}{2a} \log \left| \frac{x+a}{x-a} \right|$

c) $\frac{1}{2a} \log \left| \frac{a+x}{a-x} \right|$ d) none of these.

xii) If A is an orthogonal matrix, then det $A = |A|$ is

a) 1 only b) – 1 only

c) ± 1 d) 0.

xiii) If $x = 4t$, $y = 2t^2$, then $\frac{d^2y}{dx^2}$ is

a) 4 b) $4t$

c) $\frac{1}{4}$ d) none of these.

xiv) If e is the eccentricity of a hyperbola, then

a) $e = 0$ b) $e = 1$

c) $e > 1$ d) $e < 1$.

xv) If $A = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 0 \\ 0 & 3 \end{pmatrix}$, then AB is equal to

a) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ b) $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$

c) $\begin{pmatrix} 6 & 0 \\ 0 & 0 \end{pmatrix}$ d) $\begin{pmatrix} 0 & 0 \\ 0 & 6 \end{pmatrix}$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Solve by Cramer's rule the following simultaneous equations :

$x + y = 0$

$y + z = 1$

$z + x = -1$

3. If ω is imaginary cube root of 1, show that $a + b\omega + c\omega^2$ is a factor of

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}.$$

4. Find the points on the curve $y = x + \frac{1}{x}$ at which the tangents to the curve are parallel to x -axis.

5. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx$.

6. Evaluate $\lim_{x \to 1} (1 - x) \tan \frac{x\pi}{2}$.

7. Evaluate $\int_0^1 e^x\, dx$ as the limit of a sum.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Find the coordinates of the foci of the ellipse

$$3x^2 + 4y^2 = 12$$

b) Without expanding show that

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (b - c)(c - a)(a - b).$$

c) If $y = (\cos^{-1} x)^2$, show that

$$(1 - x^2)\ \frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2\ .$$

9. a) Find the equation of the parabola where vertex is at the origin, the axis on the y–axis and which passes through the point $(6, -3)$.

b) If $f(x) = (x - 1)\, e^x + 1$, show that $f(x)$ is an increasing function of x for all positive values of x.

c) Find the area bounded by the ellipse

$4x^2 + 9y^2 = 36$ and the x -axis.

10. a) Examine whether the following matrix

$$A = \frac{1}{3}\begin{pmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$

is orthogonal.

b) Show that all rectangles of a given perimeter, the square has the maximum area.

c) Differentiate

$\sin^{-1}\dfrac{2x}{1+x^2}$ with respect to $\tan^{-1}\dfrac{2x}{1-x^2}$.

11. a) State and prove Euler's theorem for a homogeneous function of two variables.

b) Integrate $\cos^2 x$ with respect to x.

c) Evaluate :

$$\lim_{x \to \infty} \left(\frac{1^2 + 2^2 + 3^2 + \ldots + n^2}{n^3} \right).$$

12. a) Find the equation of the ellipse whose latus rectum is 5 and whose eccentricity is $\frac{2}{3}$, the axes of the ellipse being the coordinate axes.

b) Examine the continuity of the following function $f(x)$ at $x = 0$, where $f(x)$ is defined by

$$f(x) = \frac{\sin 3x}{2x}, \text{ for } x \neq 0$$

$$= \frac{2}{3}, \text{ for } x = 0$$

c) If a function $f(x)$ is defined by

$$f(x) = \frac{1-x}{1+x}, \text{ find } f\left(f\left(\frac{1}{x} \right) \right), \text{ for } x \neq 0.$$