MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : BMN-101

PUID : 01100 (To be mentioned in the main answer script)

BASIC MATHEMATICAL COMPUTATION

Time Allotted : 3 Hours | *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) If A and B are square matrices and A^{-1} and B^{-1} exist then $(AB)^{-1}$ =

   a) $A^{-1}B^{-1}$ b) AB^{-1}

   c) $B^{-1}A^{-1}$ d) $A^{-1}B$.

   ii) The coefficient of x^{13} in the expansion of $(1-x)^5\left(1+x+x^2+x^3\right)^4$ is

   a) 0 b) -4

   c) 6 d) 4.

iii) The area of the triangle which the straight line $2x + 3y - 12 = 0$ forms with the co-ordinate axes is

a) 11 b) 14

c) 12 d) 10.

iv) The length of latus rectum of the ellipse $16x^2 + 9y^2 = 1$ is

a) $\frac{16}{3}$ b) $\frac{32}{3}$

c) $\frac{9}{4}$ d) $\frac{9}{2}$.

v) If the origin is shifted to the point (1, − 2), then the transformer equation of the curve $x^2 + 4y = 3$ is

a) $x'^2 - 2x' - 4y' + 10 = 0$

b) $x'^2 - 2x' + 4y' + 10 = 0$

c) $x'^2 + 2x' + 4y' - 10 = 0$

d) none of these.

vi) The value of $\lim_{x \to 0} \frac{4^x - 1}{x}$ is

a) 4 b) $\frac{1}{2}$

c) 2 d) none of these.

vii) The function $f(x) = \frac{x^2}{x-2}$ is

a) continuous everywhere

b) discontinuous at $x = 2$

c) discontinuous at $x = -2$

d) none of these.

viii) If $f(x)$ satisfies the conditions of Rolle's theorem in an interval $[a, b]$ then $f'(x)$ becomes zero at

a) only one point in (a, b)

b) two points in (a, b)

c) three points in (a, b)

d) none of these.

ix) $\lim_{\substack{x \to 0 \\ y \to 0}} \frac{x}{y} =$

a) 0

b) 1

c) ∞

d) does not exist.

x) The slope of the tangent to the parabola $y^2 = 4ax$ at the point $(at^2,\ 2at)$ is

a) $\frac{1}{t}$ b) t

c) $-t$ d) none of these.

xi) The value of $\int \tan 2x\, dx$ is

a) $\frac{1}{2}\log \sec 2x$ b) $\frac{1}{2}\sec^2 2x$

c) $\log \sec x$ d) $\sec^2 2x$.

xii) The value of $\int_{-1}^{1} |x|\, dx$ is

a) 0 b) 1

c) $\frac{1}{2}$ d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. How many words can be made using all the letters in the word MONDAY ?

3. Find the point on the conic $\frac{6}{r} = 1 + 4\cos\theta$ whose vectorial angle is $\frac{\pi}{3}$.

4. Find $\frac{d}{dx}\left(\frac{(x+1)^3}{x}\right)$.

5. Evaluate $\int \frac{x^2\,dx}{\sqrt{1+x^3}}$.

6. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sin x + \cos x}\,dx$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Find the equation of the circle passing through the three points (3, 4), (3, – 6) and (– 1, 2)

   b) If the eccentricities of the ellipses $\frac{x^2}{a^1} + \frac{y^2}{\beta^2} = 1$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ be same, show that $a\beta = ba$. 7 + 8

8. a) Evaluate $\lim_{x \to 0} \frac{(1+x)^{\frac{9}{2}} - 1}{x}$.

b) Find $\frac{dy}{dx}$, where $y = \log\left(\tan\frac{x}{2}\right)$.

c) Find $\frac{dy}{dx}$, where $x = a(t + \sin t)$, $y = a(1 + \cos t)$ at $t = \frac{\pi}{2}$. 5 + 5 + 5

9. a) If $u = x^2y + y^2z + z^2x$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = (x+y+z)^2$

b) Verify Euler's theorem (1st order) for the function $u(x,y) = x^3 - y^3 + 3x^2y + 3xy^2$. 7 + 8

10. a) Evaluate $\int \frac{dx}{x^2 + 4x + 5}$.

b) Evaluate $\lim_{x \to 0} \frac{\log x - 1}{x - e}$.

c) Evaluate $\int x^2 e^x \, dx$. 5 + 5 + 5

11. a) Verify Rolle's theorem for the function $f(x) = x^2 - 5x + 6$ in [2, 3].

b) A room has 6 doors. In how many way can a man enter the room through one and come out through a different door ?

c) Find $\frac{dy}{dx}$ when $x^y + y^x = 1$. 6 + 4 + 5