Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/BCA/SEM-1/BM-101/2013-14

2013

MATHEMATICS

Time Allotted : 3 Hours   *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words

as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

i)   If $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2$, then the equation

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of

straight lines if

a) $\Delta > 0$ b) $\Delta < 0$

c) $\Delta = 0$ d)   none of these.

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ii) If the matrix $\begin{pmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ \lambda & -3 & 0 \end{pmatrix}$ is singular then the

value of λ is

a) 0

b) 4

c) 2

d) −1.

iii) If A be a matrix whose inverse exists then which of the

following is not true ?

a) $(A^T)^{-1} = (A^{-1})^T$

b) $A^{-1} = (\det A)^{-1}$

c) $(A^2)^{-1} = (A^{-1})^2$

d) None of these.

iv) $\dfrac{\partial}{\partial x}(e^{xy}) =$

a) e^{xy}

b) xe^{xy}

c) ye^{xy}

d) none of these.

v) The degree of the function $f(x,y) = \tan^{-1}\dfrac{y}{x}$ is

a) 1

b) 0

c) 2

d) none of these.

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vi) The inverse of the matrix $\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$ is

    a) $\dfrac{1}{3}\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 1 & -2 \\ -1 & 1 \end{pmatrix}$

    c) $\dfrac{1}{3}\begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix}$ d)   none of these.

vii) The value of $\displaystyle\int \dfrac{dx}{x \log x}$ is

    a) $\log |x| + c$ b) $\log |\log x| + c$

    c) $x \log |x| + c$ d)   none of these.

viii) If α, β and γ be the roots of the equation $x^3 + 7x - 2 = 0$ then $\sum \alpha^2 =$

    a)   0   b)   14

    c) -14 d)   4.

ix) Which of the following is a null set ?

    a) $A = \{0\}$

    b) $A = \{\phi\}$

    c) $A = \{x : x \text{ is an integer } \& \ 1 < x < 2\}$

    d)   None of these.

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x) The value of $\underset{x \to 0}{Lim} \dfrac{\sin x}{x}$ (where x is radian) is

    a)   1   b)   0

    c) ∞ d) -1.

xi) The conic $\dfrac{l}{r} = 1 - e\cos\theta$ represents a parabola if

    a) $e = 1$ b) $e > 1$

    c) $e < 1$ d)   none of these.

xii) What is the value of the following limit ?

$$\underset{x \to 0}{Lim}(1+x)^{1/x}$$

    a)   1   b) e

    c)   0   d)   none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Evaluate the integral $\displaystyle\int_{0}^{\frac{\pi}{2}} \dfrac{\sin x}{\sin x + \cos x}\,\mathrm{d}x$.

3. Express $\begin{bmatrix} -3 & 4 & 1 \\ 2 & 3 & 0 \\ 1 & 4 & 5 \end{bmatrix}$ as the sum of a symmetric and a skew-

symmetric matrix.

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4. If $u = \tan^{-1} \dfrac{x+y}{\sqrt{x}+\sqrt{y}}$, then show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{1}{4}\sin 2u$.

5. Solve the equation $x^3 - 9x^2 + 14x + 24 = 0$ two of whose roots are in the ratio 3 : 2.

6. Prove that the set of real numbers of the form $a + b\sqrt{2}$ where a and b are rational numbers, forms a field under addition and multiplication.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following.   3 × 15 = 45

7. a) State Decartes' rule of sign. Using this rule find the nature of the root of the equation $x^4 - 7x^3 + 21x^2 - 9x + 21 = 0$.   5

   b) Solve the following system of linear equations by Cramer's rule :   5

   $x - y + 2z = 1$, $x + y + z = 2$, $2x - y + z = 5$.

   c) If by a transformation of rectangular axis to another with same origin the expression $ax + by$ changes to $a'x' + b'y'$, prove that $a^2 + b^2 = a'^2 + b'^2$.   5

8. a) If G be a group such that $(ab)^2 = a^2 b^2 \ \forall a,b \in G$, show that the group G is Abelian.   5

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b) Show that $\int\limits_0^1 \dfrac{\log(1+x)}{1+x^2}\,dx = \dfrac{\pi}{8}\log 2$. 　　5

c) If $y = e^{-x}\sin x$, then show that $y_4 + 4y = 0$. 　　5

9. a) Show that the matrix $A = \dfrac{1}{3}\begin{pmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ is orthogonal

and hence find A^{-1}. 　　4

b) If $A = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$, then show that $A^2 - 2A + I_2 = O_2$. Hence

obtain A^{-1} and also find A^{100}. 　　4

c) Reduce the following equation to the canonical form and determine the nature of the conic represented by it :

$8x^2 - 12xy + 17y^2 + 16x - 12y + 3 = 0$. 　　7

10. a) Solve the equation $x^3 - 3x^2 + 12x + 16 = 0$ by Cardan's method. 　　6

b) Prove that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$. 　　4

c) If α, β, γ are the three roots of $x^3 + px^2 + qx + r = 0$, obtain the value of $\sum (\alpha - \beta)^2$. 　　5

11. a) State Rolle's theorem. Examine whether Rolle's theorem is applicable or not for the function $f(x) = 1 - |x-1|, \forall x \in [0,2]$.

b) If $u = \dfrac{y}{z} + \dfrac{z}{x} + \dfrac{x}{y}$, then prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 0$.

c) Find for what values of x, the following expression is maximum and minimum respectively :

$2x^3 - 21x^2 + 36x - 20$ 5 + 5 + 5

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