

CS/BCA/Odd/Sem-1st/BM-101/2014-15

**BM-101**

**MATHEMATICS**

Time Allotted: 3 Hours

Full Marks: 70

*The questions are of equal value.  
The figures in the margin indicate full marks.  
Candidates are required to give their answers in their own words as far as practicable.*

**GROUP A**  
**(Multiple Choice Type Questions)**

1. Answer any *ten* questions. 10×1 = 10
- (i) An element  $x$  in a ring  $R$  is a zero divisor if
- (A)  $x \cdot b = 0$
  - (B)  $x \cdot b = 0$ , for some non zero element in  $R$
  - (C)  $x \cdot b \neq 0$ , for all elements  $b$  in  $R$
  - (D) none of these
- (ii) If  $A = \{2, 4, 6\}$  and  $B = \{1, 3, 5, 7\}$  then  $A \cup B$  is
- (A)  $\{0\}$
  - (B)  $\{1, 2, 3, 4, 5, 6, 7\}$
  - (C)  $\{1, 2, 4, 5, 6, 7\}$
  - (D)  $\{0, 2\}$
- (iii) The polar form of the equation  $x^2 + y^2 - 8y = 0$  is
- (A)  $r = 8\cos\theta$
  - (B)  $r = 8\sin\theta$
  - (C)  $r^2 = 8\cos\theta$
  - (D) none of these
- (iv) The diagonal elements of a real skew symmetric matrix are
- (A) 1
  - (B) -1
  - (C) 2
  - (D) 0
- (v) If  $\alpha, \beta, \gamma$  are the roots of the equation  $x^3 - 3x^2 + 6x - 2 = 0$  then,  $\alpha + \beta + \gamma$  is
- (A) 0
  - (B) 1
  - (C) 3
  - (D) -2

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(vi) The value of  $t$  for which the matrix  $\begin{bmatrix} 2 & 0 & 1 \\ 5 & t & 3 \\ 0 & 3 & 1 \end{bmatrix}$  is singular is

- (A)  $\frac{-3}{2}$                       (B)  $\frac{3}{2}$                       (C) 2                      (D) -2

(vii) The function  $f(x) = |x|$  then

- (A) continuous and differentiable at  $x = 0$   
(B) continuous everywhere but differentiable at  $x = 0$   
(C) discontinuous and not differentiable at  $x = 0$   
(D) none of these

(viii) Which of the following function obeys Rolle's theorem in  $[0, \pi]$

- (A)  $x$                       (B)  $\sin x$                       (C)  $\cos x$                       (D)  $\tan x$

(ix) By 3<sup>rd</sup> order Maclaurin's theorem we have  $\sin x = f(x) - \frac{x^3}{6} \cos \theta x$ , then  $f(x)$  equal to

- (A)  $x^2$                       (B)  $-x^2$                       (C)  $x$                       (D)  $-x$

(x) If  $f(x, y) = x^2y$  then  $df$  equal to

- (A)  $2x^2 dx + dy$                       (B)  $x - 2 dy$                       (C)  $x + dy$                       (D)  $2xy dx + x^2 dy$

(xi)  $f(x, y) = |x| + |y|$  then  $f_x(0, 0)$  equal to

- (A) 0                      (B) 1                      (C) does not exist                      (D) none of these

(xii) The value of  $\int_1^2 \frac{e^{\log x}}{x} dx$

- (A) 1                      (B) -1                      (C) 2                      (D) 0

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**GROUP B**  
(Short Answer Type Questions)

Answer any *three* questions.

3×5 = 15

2. If  $\alpha, \beta$  are the roots of the equation  $x^2 - px + q = 0$  then find the equation whose roots are  $\frac{1}{\alpha}$  and  $\frac{1}{\beta}$ .
3. Solve the following equations by matrix inversion method  
 $x + y + z = 2, x - y + 2z = 6, 3x + 5y + 7z = 14$
4. Give the definition of commutative group and show that  $\{1, \omega, \omega^2\}$  where  $\omega^3 = 1$  forms a commutative group w.r.t. multiplication.
5. If  $y = \cos(m \sin^{-1} x)$  then prove that  $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$
6. If  $u = \tan^{-1} \frac{x^2 - y^2}{x - y}$ , show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \sin 2u$ .

**GROUP C**  
(Long Answer Type Questions)

Answer any *three* questions.

3×15 = 45

7. (a) In a class of 50 students, 15 read Physics, 20 read Chemistry and 20 read Mathematics, 3 read Physics and Chemistry, 6 read Chemistry and Mathematics and 5 read Physics and Mathematics, 7 read none of the subjects. How many students read all the subjects? 7
- (b) Discuss the nature of the conic represented by  $3x^2 - 8xy - 3y^2 + 10x - 13y + 8 = 0$  by reducing to its canonical form. 8
8. (a) Apply Descartes's rule of sign to show that the equation  $x^4 + 2x^2 - 7x - 5 = 0$  has two real roots and two non real roots. 5
- (b) Verify Rolle's theorem for the function  $f(x) = |x|, -1 \leq x \leq 1$  5

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- (c) Discuss the continuity of the following function  $f(x) = x - [x]$ , where  $[x]$  denotes the greatest integer not greater than  $x$ . 5
9. (a) Using the mean value theorem prove the following inequalities  $x < \sin^{-1}x < \frac{x}{\sqrt{1-x^2}}$  if  $0 < x < 1$  6
- (b) Show that  $z = \log\{(x-a)^2 + (y-b)^2\}$  satisfies the relation  $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$  except at  $(a, b)$  6
- (c) Evaluate  $\int \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$  3
- 10.(a) Solve  $x^3 - 9x + 28 = 0$  using Cardan's method. 6
- (b) Evaluate  $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$  4
- (c) Find the nature of the conic  $\frac{8}{r} = 4 - 5\cos\theta$  5
- 11.(a) Find  $\frac{dy}{dx}$  when  $x = y \log(xy)$  5
- (b) Give the definition of a ring with two binary composition. Let  $H$  be the set of all matrices  $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : ad - bc = 1 \right\}$  5
- Prove that  $H$  forms a non-commutative group with respect to matrix multiplication.
- (c) If by a transformation of motion of co-ordinate axis, the expression  $ax^2 + 2hxy + by^2$  changes into  $a'x'^2 + 2h'x'y' + b'y'^2$  then show that  $a + b = a' + b'$  5