CS/BCA/ODD SEM/SEM-1/BM-101/2016-17

Maulana Abul Kalam Azad University
of Technology, West Bengal

Utech

MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : BM-101
MATHEMATICS

Time Allotted : 3 Hours   *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :   10 × 1 = 10

    i)   If A and B are square materics and A^{-1} and B^{-1} exist then $(AB)^{-1}$ =

    a) $A^{-1}B^{-1}$ b) AB^{-1}

    c) $B^{-1}A^{-1}$ d) $A^{-1}B$

    ii) Which of the following is a null set ?

    a) $A = \{ 0 \}$

    b) $A = \{ x : x$ is an integer and $1 < x < 2 \}$

    c) $A = \{ \phi \}$

    d)   None of these.

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iii) What is the value of $\lim\limits_{x \to 0} (1-x)^{\frac{1}{x}}$?

a)   1

b)   0

c)   e

d) $\dfrac{1}{e}$.

iv) If α, β and γ are the roots of the equation $2x^3 - 4x + 10 = 0$ then $\sum \alpha\beta =$

a)   10

b)   8

c)   –4

d)   –2.

v) If $f(x) = \dfrac{x}{|x|}$; when $x \neq$

$= 0$ when $x = 0$

a) $f(x)$ is continuous at $x = 0$

b) $f(x)$ is continuous, but not differentiable at $x = 0$

c) $f(x)$ is discontinuous at $x = 0$

d)   none of these.

vi) The nature of the conic

$4x^2 + 4xy + y^2 - 12x - 6y + 5 = 0$ is

a)   ellipse

b)   hyperbola

c)   parabola

d)   pair of parallel straight lines.

vii) The trace of a real skew-symmetric matrix is

   a) 0   b) 1

   c) −1   d) none of these.

viii) If $y = cx^3$, then the value of $\dfrac{d^4y}{dx^4}$

   a) c b) $3c$

   c) 0   d) 6.

ix) If $f(x) = \int\limits_{0}^{x} t \, \sin t \, dt$ then $f'(x)$ is equal to

   a) $\cos x + x \sin x$ b) $x \sin x$

   c) $x \cos x$ d) none of these.

x) $f(x, y) = \dfrac{\sqrt{y} + \sqrt{x}}{\sqrt{xy} + x}$ is a homogeneous function of

   degree

   a) $\dfrac{1}{2}$ b) $-\dfrac{1}{2}$

   c) 1   d) 2.

xi) If A is a 3rd order square matrix then $|5A| =$

   a) $5|A|$ b) $5|A|^3$

   c) $125|A|$ d) $125|A|^3$.

xii) The equation $r = 3 \sin \theta + 4 \cos \theta$ represents

   a) a parabola   b) an ellipse

   c) a straight line   d) a circle.

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GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Prove that every square matrix can be expressed as the sum of a skew-symmetric and symmetric matrix.

3. If $y = (\sin^{-1} x)^2 + (\cos^{-1} x)^2$ then   prove   that

   $(1 - x^2) y_2 - x y_1 = 4$.

4. If $u = \dfrac{x^2 - y^2}{\sqrt{x - y}}$, then find the value of $x \dfrac{\partial u}{\partial x} + y \dfrac{\partial u}{\partial y}$.

5. Solve the following system of linear equations by Cramer's Rule or Matrix Inversion method :

   $2x + 5y + 3z = 9$

   $3x + y + 2z = 3$

   $x + 2y - z = 6$

6. What does the equation $x^2 - 3xy + 3y^2 + 7x - 18y + 32 = 0$ become when the origin is moved to the point (4, 5) and the axes are turned through an angle of 45° ?

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GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Solve the equation $x^3 + 9x^2 + 15x - 25 = 0$ by Cardan's method.

    b) For any three sets

        A, B, C show that $A - (B - C) = (A - B) \cup (A \cap C)$.

    c) Show that $\int\limits_0^\infty \dfrac{dx}{(x+1)(x+2)} = \log 2$. $7 + 3 + 5$

8. a) If $2x = y^{\frac{1}{m}} + y^{\frac{-1}{m}}$, then prove that

        $\left(x^2 - 1\right)y_{n+2} + (2n+1)xy_{n+1} + \left(n^2 - m^2\right)y_n = 0$.

    b) If $z = f(z, y)$ where $x = e^u \cos v$, $y = e^u \sin v$

        then show that $y\dfrac{\partial z}{\partial u} + x\dfrac{\partial z}{\partial v} = e^{2u}\dfrac{\partial z}{\partial y}$.

    c) Give the definition of a ring with two binary compositions. Let H be the set of all matrices $\left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix}; ad - bc = 1\right\}$.

        Prove that H forms a non-commutative group with respect to matrix multiplication. $4 + 4 + 7$

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9. a) If by a transformation of motion of co-ordinate axis, the expression $ax^2 + 2hxy + bh^2$ changes into $AX^2 + 2HXY + BY^2$ then show that $a + b = A + B$ and $ab - h^2 = AB - H^2$.

b) Find the quotient and remainder when $f(x) = x^4 - 7x^3 + x^2 - 11$ is divided by $(x - 2)$.

c) Find the maxima and minima of $x^3 - 6x^2 + 9x - 8$.

5 + 5 + 5

10. a) Prove that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

b) In a survey of 320 persons, number of persons taking tea is 210, taking milk is 100 and coffee is 70. Number of persons who take tea and milk is 50, milk and coffee is 30, tea and coffee is 50. The number of persons all three together is 20. Find the number of people who take neither tea not coffee nor milk.

c) If $x^m y^n = (x+y)^{m+n}$ then show that $\dfrac{dy}{dx} = \dfrac{y}{x}$.

5 + 5 + 5

11. a) If G be a group such that $(ab)^2 = a^2 b^2$ for all $a, b \in G$

   show that the group G is Abelian.

   b) Apply Descartes' rule of signs to find the nature of the

   roots of $x^4 - 3x^2 + 8x + 12 = 0$

   c) A function $f(x)$ is defined as follows :

   $$f(x) = \begin{cases} -x & when & x \leq 0 \\ x, & when & 0 < x \leq 1 \\ 2-x, & when & x \geq 1 \end{cases}$$

   Show that f (x) is continuous at $x = 0$ and $x = 1$.

   $$5 + 5 + 5$$