

# CS/BCA/SEM-1 /BM-101/2009-10 2009 MATHEMATICS 

Time Allotted : 3 Hours
Full Marks : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

## GROUP - A

( Multiple Choice Type Questions )

1. Choose the correct alternatives for any ten of the following :

$$
10 \times 1=10
$$

i) The value of $\lim _{x \notin 2} \frac{x^{2}-4}{x-2}$ is
a) 1
b) 4
c) 0
d) 2 .
ii) The value of $\int^{2} \mathrm{~d} x$ is equal to 1
a) 1
b) 2
c) 3
d) 0 .
iii) If $x=-1$ is a root of the equation $x^{2}-x-k=0$, then the value of $k$ is
a) 1
b) 0
c) $\sqrt{2}$
d) 2 .

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iv) If $\alpha, \beta, \gamma$ be the roots of the equation $x^{3}-3 x^{2}+6 x-2=0$, then $\alpha+\beta+\gamma$ is
a) 2
b) 1
c) 3
d) none of these.
v) If $A\{1,2,3,4\}$ and $B=\{2,4,6\}$, then $A \Delta B$ is
a) $\{1,2\}$
b) $\{1,2,3,6\}$
c) $\quad\{1,3,6\}$
d) $\{6\}$.
vi) What is the order of the matrix $B$, if $\left[\begin{array}{lll}3 & 4 & 2\end{array}\right]$ $B=\left[\begin{array}{lllll}2 & 10 & 3 & 6 & 9\end{array}\right] ?$
a) $1 \propto 5$
b) $1 \infty 3$
c) $3 \propto 5$
d) $5 \propto 3$.
vii) The degree of the polynomial

$$
f(x)=\left(x^{2}+x-2\right) /(x-1) \text { is }
$$

a) 0
b) 1
c) 2
d) 3 .
viii) If $y=\log x^{2}$, the value of $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}$ is
a) $\frac{2}{x^{3}}$
b) $-\frac{2}{x^{2}}$
c) $\frac{2}{x}$
d) $2 x$.
ix) The value of $t$ for which the matrix
 singular, is
a) $-\frac{3}{2}$
b) 2
c) $\frac{3}{2}$
d) -2 .
$\mathrm{x}) \lim _{x \notin 0}(1+x)^{1 / x}$ is equal to
a) 1
b) $e$
c) •
d) 0 .
xi) If $\alpha, \beta, \gamma$ be the roots of the equation

$$
x^{3}-3 x^{2}+6 x-2=0, \text { then } \sum \alpha \beta \text { is }
$$

a) 3
b) 6
c) 2
d) none of these.
xii) If $A\{1,2,3\}$ and $B=\{2,3,6\}$, then $A \cup B$ is
a) $\{1,2,3\}$
b) $\{2,3\}$
c) $\{1,2,3,6\}$
d) none of these.

## GROUP - B

( Short Answer Type Questions )
Answer any three of the following. $3 \times 5=15$
2. Evaluate the integral $\int^{\pi / 2} \frac{\sin x}{\sin x+\cos x} \mathrm{~d} x$. 0

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3. If $u=\log r$ and $r^{2}=x^{2}+y^{2}+z^{2}$, prove that

$$
r^{2}\left[\frac{\partial^{2} u}{\partial x^{2}}+\frac{\partial^{2} u}{\partial y^{2}}+\frac{\partial^{2} u}{\partial z^{2}}\right]=1
$$

4. In a survey of 320 persons, number of persons taking tea is 210, taking milk is 100 and coffee is 70 . Number of persons who take tea and milk is 50 , milk and coffee is 30 , tea and coffee is 50 . The number of persons taking all three together is 20 . Find the number of people who take neither tea nor coffee nor milk.
5. Express $\left[\begin{array}{rrr}-3 & 4 & 1 \\ 2 & 3 & 0 \\ 1 & 4 & 5\end{array}\right]$ as a sum of a symmetric and a skew-symmetric matrix.
6. If $\alpha, \beta, \gamma$ be the roots of the equation $x^{3}+2 x^{2}+3 x+4=0$, then find the equation whose roots are

$$
1+\frac{1}{\alpha}, \quad 1+\frac{1}{\beta} \quad \text { and } \quad 1+\frac{1}{\gamma} .
$$

## GROUP - C

( Long Answer Type Questions )
Answer any three of the following. $3 \times 15=45$
7. a) Verify whether the matrix $\left[\begin{array}{rr}\cos \theta & \sin \theta \\ -\sin \theta & \cos \theta\end{array}\right]$ is orthogonal.
b) Solve the following system of linear equations by using Cramer's Rule :

$$
\begin{aligned}
& 2 x+5 y+3 z=9 \\
& 3 x+y+2 z=3 \\
& x+2 y-z=6
\end{aligned}
$$

c) If $A=\left[\begin{array}{lll}1 & 2 & 3 \\ 2 & 3 & 1\end{array}\right]$ and $B=\left[\begin{array}{cc}1 & 1 \\ 2 & -2 \\ 3 & 3\end{array}\right]$, find $A B$.
d) Show that $\left[\begin{array}{rr}\cos \theta & -\sin \theta \\ \sin \theta & \sin \theta\end{array}\right] \infty\left[\begin{array}{rr}\cos \theta & -\sin \theta \\ \sin \theta & \sin \theta\end{array}\right]$

$$
\left[\begin{array}{cc}
= & -\sin \theta(\sin \theta+\cos \theta) \\
\cos 2 \theta & 0
\end{array}\right]
$$

$$
2+5+4+4
$$

8. a) Evaluate any two :
i) $\lim _{x \notin 0} \frac{\tan 2 x-x}{3 x-\sin x}$
ii) $\lim _{x \notin 0} \frac{x \log \sqrt{1+x}}{\sin ^{2} x}$
iii) $\lim _{x \notin a} \frac{1-\cos (x-a)}{(x-a)^{2}}$
b) Evaluate $\int_{0}^{\pi / 2} x^{2} \sin x \mathrm{~d} x$.
c) Differentiate $\frac{x^{3}}{\left(1+x^{3}\right)}$ with respect to $x^{4}$.
9. a) If $A=\{a, b, c, d, e\}, B=\{c, a, e$ $C=\{b, e, f, g\}$, then show that

$$
(A \cup B) \cap C=(A \cap C) \cup(B \cap C)
$$

b) If $A=\left[\begin{array}{ccc}1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0\end{array}\right]$, then find $A^{2}$ and show that

$$
A^{2}=A^{-1} .
$$

c) Find the maxima and minima of $x^{3}-6 x^{2}+9 x-8$.

$$
5+5+5
$$

10. a) Determine whether the function

$$
\begin{aligned}
f(x, y) & =\frac{x y}{x^{2}+y^{2}} & & \text { if }(x, y) \neq(0,0) \\
& =0 & & \text { if }(x, y)=(0,0)
\end{aligned}
$$

is continuous at the origin.
b) Apply Descartes' rule of signs to find the nature of roots of the equation

$$
x^{4}+2 x^{2}+x-12=0
$$

c) State Cauchy's mean value theorem.
11. a) Find the value of ' $a$ ' and ' $b$ ' for which the system of equations

$$
\begin{aligned}
& x+2 y+z=1 \\
& 2 x+y+3 z=b \\
& x+a y+3 z=b+1
\end{aligned}
$$


has (i) unique solution, (ii) many solutions.
b) Solve the following system of equations by matrix inversion method

$$
\begin{aligned}
& x+y+z=6 \\
& x-2 y+z=0 \\
& 2 x-y+z=3
\end{aligned}
$$

c) Find out the rank of the matrix $\left[\begin{array}{rrr}2 & -4 & 6 \\ 2 & 3 & -1 \\ 3 & 1 & 2\end{array}\right]$.
12. a) If $u=\cos ^{-1}\{(x+y) / \sqrt{x}+\sqrt{y}\}$, then show that $x \cdot \frac{\partial u}{\partial x}+y \cdot \frac{\partial u}{\partial y}+\frac{1}{2} \quad \cot u=0$
b) If $P S Q$ be a focal chord of a conic with focus $S$ and semi latus rectum $L$, then prove that

$$
1 / S P+2 / S Q=2 / L
$$

c) Find the point on the conic $6 / r=1+4 \cos \theta$ whose vertical angle is $\pi / 3$.

$$
8+4+3
$$

