Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/BCA/SEM-1/BM-101/2010-11

2010-11
MATHEMATICS

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) $\lim_{x \to 0} (1+x)^{1/x} = ?$

   a) 1

   b) 0

   c) $\frac{2}{3}$

   d) e.

   ii) If α, β, χ be the roots of the equation $x + yn = 2$ then $\Sigma x^2 =$

   a) 0 b) 14

   c) – 14 d) 4.

iii) An element x in a ring R is zero divisor if

a) $x \cdot b = 0$

b) $x \cdot b = 0$, for some non zero element b in R

c) $x \cdot b \neq 0$, for all element b in R

d) none of these.

iv) The value of $\int_{-1}^{2} |x| \, dx$ is

a) 3 b) 5

c) $\frac{5}{2}$ d) 0.

v) The value of $\frac{d}{dx}(\log_e x)$ is equals to

a) $\frac{1}{x}$ b) $\log\left(\frac{1}{x}\right)$

c) $\left(\frac{1}{n}\right)\log_a e$ d) $a \log e$.

vi) If $A = \{ 2, 4, 6 \}$ and $B = \{ 1, 3, 5, 7 \}$, then $A \cup B$ is

a) $\{ 0 \}$ b) $\{ 1, 2, 3, 4, 5, 6, 7 \}$

c) $\{ 1, 2, 4, 5, 6, 7 \}$ d) $\{ 0, 2 \}$.

vii) If A is a square matrix then

a) $A + A^T$ is symmetric

b) $A + A^T$ is skew symmetric

c) $A - A^T$ is symmetric

d) $A - A^T$ is skew symmetric.

viii) The matrix $A = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$ is on

a) orthogonal matrix b) idempotent matrix

c) identity matrix d) none of these.

ix) If y = 2 at and $x = at^2$, then $\frac{dy}{dx}$ at t = 1 is

a) 1 b) $2a$

c) – 1 d) $2a^2$.

x) The polar form of the equation $x^2 + y^2 - 8y = 0$ is

a) $r = 8 \cos \theta$ b) $r = 8 \sin \theta$

c) $r^2 = 8 \cos \theta$ d) none of these.

xi) If A = { 1, 2, 3, 4, 8 }, B = { 2, 4, 6, 7 } then $A \Delta B$ is

a) { 2, 4 }

b) { 1, 2, 3, 4, 6, 7, 8 }

c) ϕ

d) { 1, 3, 6, 7, 8 }.

xii) The diagonal elements of a real skew-symmetric matrix are

a) 1 b) – 1

c) 2 d) 0.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. A function $f(x)$ is defined as follows

$f(x) = x^2$ when $0 < x < 1$

$= x$ when $1 \le x < 2$

$= 2 - x$ when $2 \le x < 3$

Show that the $f(x)$ is continuous at $x = 2$.

3. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$.

4. If α, β, γ be the roots of the cubic $x^3 + px + q = 0$, then find the equation whose roots are

$\frac{\beta+\gamma}{\alpha^2}, \frac{\gamma+\alpha}{\beta^2}, \frac{\alpha+\beta}{\gamma^2}$.

5. Prove that the ring of matrices of the form $\begin{bmatrix} x & y \\ -y & x \end{bmatrix}$ of real number is a field.

6. In a survey concerning the smoking habits of consumers it was found that 55% smoke cigarette-*A*, 50% smoke cigarette-*B*, 42% smoke cigarette-*C*, 28% smoke cigarette-*A* & *B*, 20% smoke cigarette-*A* & *C*, 12% smoke cigarette-*B* & *C* and 10% smoke all the three cigarette. What percentage do not smoke ?

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

7. a) If $y = \sin\left(m \sin^{-1} x\right)$, then show that $\left(1-x^2\right)y_{n+2} - (2n+1)xy_{n+1} + \left(m^2 - n^2\right)y_n = 0$.

b) If α, β, γ are the 3 roots of $x^3 + px^2 + qx + r = 0$ obtain the value of $\sum(\alpha - \beta)^2$.

c) Evaluate $\int \frac{1}{x^2} e^{1/x}\, dx$.

8. a) If $u = \frac{y}{z} + \frac{z}{x} + \frac{z}{y}$ then prove that, $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.

b) If by a rotation of rectangular co-ordinate axes without change of origin expressions $ax + by$ and $cx + dy$ are transformed into $a' x' + b' y'$ and $c' x' + d' y'$. Show that $a' d' - b' c' = ad - bc$.

c) Reduce the following equation to its canonical form and determine the nature of the conic represented by it :

$3x^2 - 8xy - 3y^2 + 10x - 13y + 18 = 0$

9. a) Evaluate

$$\lim_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+.....+\frac{n}{n^2+n^2}\right].$$

b) Using mean value theorem prove the following inequality :

$$x\left\langle \sin^{-1}x\left\langle \frac{x}{\sqrt{1-x^2}}\right.\right., \text{ if } 0 < x < 1$$

c) Expand sin x in power of x in infinite series.

10. a) Solve the equation by Cardan's method :

$$2x^3+3x^2+3x+1$$

b) Evaluate

$$\int\frac{x^2\,dx}{\left(x^2+a^2\right)\left(x^2+b^2\right)}$$

c) If $y=x^{x-1}\log x$, show that $y_x=\frac{(x-1)!}{x}$.

11. a) Prove that $|A \cup B| = |A| + |B| - |A \cap B|$ where A and B are two non-empty sets.

b) If $A = \{ a, b, c, d \}$ $B = \{ b, c, p, q \}$, then find out $A \times B$, $B \times A$ and $A \Delta B$.

c) Define power set. Find the power set of $\{ a, b, c \}$.