



- b) If $y = \sin^{-1} x$, then prove that $(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - n^2 y_n = 0$
- c) In the mean value theorem $f(x + h) = f(x) + h f'(x + \theta h)$, if $f(x) = px^2 + qx + r$ ($p \neq 0$), then show that $\theta = \frac{1}{2}$.
8. a) Reduce the equation $3x^2 + 2xy + 3y^2 - 16x + 20 = 0$ into canonical form and hence determine the nature of the conic.
- b) Find the nature of the conic $\frac{8}{r} = 4 - 5 \cos \theta$.
- c) Expand e^x in ascending powers of x by Taylor's series.
9. a) Solve using Carden's method : $x^3 - 9x + 28 = 0$.
- b) If by a transformation of motion of co-ordinate axes, the expression $ax^2 + 2hxy + by^2$ changes into $a'x'^2 + 2h'x'y' + b'y'^2$, then show that $ab - h^2 = a'b' - h'^2$.
- 8 + 7
10. a) Solve the equations by matrix inversion method :
- $$\begin{aligned} x + y + z &= 4 \\ 2x - y + 3z &= 1 \\ 3x + 2y - z &= 1 \end{aligned}$$
- b) If $u = \tan^{-1} \frac{x^2 + y^2}{x + y}$, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \sin 2u$.
- c) Evaluate : $\int \frac{x - 1}{(x - 2)(x - 3)} dx$
11. a) Give the definition of a ring with two binary composition. Let H be the set of all matrices $\left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : ad - bc = 1 \right\}$. Prove that H forms a non-commutative group.
- b) Apply Descarte's rule of sign to find the nature of the roots of the given equation : $x^4 + qx^2 + rx - s = 0$ (where q, r, s being positive).
- c) Evaluate : $\lim_{n \rightarrow \infty} \left[\frac{1}{n^2 + 1^2} + \frac{1}{n^2 + 2^2} + \dots + \frac{1}{n^2 + n^2} \right]$