MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : BMN-201(N)

ADVANCED MATHEMATICAL COMPUTATION

Time Allotted : 3 Hours | *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) A monotonic and bounded sequence is

   a) convergent b) divergent

   c) oscillatory d) none of these.

   ii) $y = ae^{x} + be^{-x}$ satisfies the differential equation

   a) $y_2 + y_1 - y = 0$ b) $y_2 - y = 0$

   c) $y_2 + y = 0$ d) $y_2 + y_1 + y = 0$.

iii) The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{2}{3}} - 3\frac{dy}{dx} = 4$ are

a) 4, 2 b) 1, 4

c) 2, 4 d) 1, 2.

iv) If $z_1 = 2 + i$, $z_2 = 1 + 3i$ then $\text{Re}(z_1 - z_2) =$

a) 1 b) i

c) $2i$ d) 2.

v) If $A = \{ 2, 4, 6 \}$ and $B = \{ 1, 3, 5, 7 \}$ then $A \cup B$ is

a) $\{0\}$ b) $\{1, 2, 3, 4, 5, 6, 7\}$

c) $\{1, 2, 4, 5, 6, 7\}$ d) $\{0, 2\}$

vi) If a be an element of a group (G, o), then which of the following is *not true* ?

a) $o(a) = o(a^{-1})$

b) If $o(a) = n$, then $a, a^2, a^3, \ldots, a^n$ are distinct elements of G

c) If $o(a) = n$, then $o(a^p) = n$ iff p is divisor of n.

d) If $o(a) = n$, and $a^m = e$, n is divisor of m.

vii) If ω be a root of the equation $x^3 = 1$, then which of the following is *true* ?

a) $\omega^3 = 1$ and $1 - \omega + \omega^2 = 0$

b) $\omega^3 = -1$ and $1 - \omega + \omega^2 = 0$

c) $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$

d) $\omega^3 = -1$ and $1 + \omega + \omega^2 = 0$.

viii) The series $\int_{n=1}^{\infty} \frac{1}{\sqrt{n} + \sqrt{n+1}}$ is

a) convergent b) divergent

c) oscillatory d) none of these.

ix) The order of the differential equation whose general solution is $y = a(x - a)^2$, where a is an arbitrary constant is

a) 1 b) 2

c) 3 d) none of these.

x) $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ is convergent if

a) sequence $\{a_n\}$ is monotonic decreasing

b) $\lim_{n \to \infty} a_n = 0$

c) both (a) and (b)

d) none of these.

xi) $\frac{\alpha}{r}$, α, α, r be the roots of $x^3 - px^2 + qx - r = 0$, then value of α is

a) $\frac{p}{q}$ b) $\frac{q}{p}$

c) pq d) pr.

xii) Number of inverse element in a group of n elements is

a) more than one b) exactly one

c) at most n d) exactly n.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Prove that, $\sin\left(i \log \frac{a - ib}{a + ib}\right) = \frac{2ab}{a^2 + b^2}$.

3. Solve $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$.

4. Solve $(D^2 - 2D + 1)y = xe^2$, $D \equiv \frac{d}{dx}$.

5. State D' Alembert's ratio test. Test the convergence of the series $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$.

6. If the equation $x^4 + ax^3 + bx^2 + cx + d = 0$ has three equal roots, the show that each of them is equal to $\frac{6c - ab}{3a^2 - 8b}$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Find the values of $(1 + i)^{1/5}$.

   b) Discuss the convergence of the series $\sum_{n=1}^{\infty} \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{\frac{3}{2}}}$.

   c) Apply Descartes' rule of signs to find the nature of the roots of the equation $x^4 + 16x^2 + 7x - 11 = 0$.

   $5 + 5 + 5$

8. a) If α, β, γ be the roots of the equation $x^3 + 2x^2 + 3x + 4 = 0$, then find the equation whose roots are $1 + \frac{1}{\alpha}, 1 + \frac{1}{\beta}, 1 + \frac{1}{\gamma}$.

b) Solve $(xy^2 - e^{1/x^3})\,dx - x^2y\,dy = 0$.

c) Solve $y = px + \sqrt{a^2p^2 + b^2}$, where $p \equiv \frac{dy}{dx}$. 5 + 5 + 5

9. a) Examine the convergence of the following series for different values of x :

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^2+1}}\, x^n$$

b) Test the convergence of the following series :

$$1 + \frac{1}{2}\cdot\frac{x^2}{4} + \frac{1.3.5}{2.4.6}\cdot\frac{x^4}{8} + \frac{1.3.5.7.9}{2.4.6.8.10}\cdot\frac{x^6}{12} + \ldots\ldots\infty$$

c) Show that the map $f : Q \to Q$ defined by $f(x) = 3x + 2$ is one-one onto, where Q is the set of rational numbers. 5 + 5 + 5

10. a) Let $(F, +, .)$ be a field and $a, b \in F$ with $b \neq 0$. Then show that $a = 1$ when $(ab)^2 = ab^2 + bab - b^2$.

b) Solve : $\frac{d^2y}{dx^2} - 9y = xe^{3x}$.

c) Solve : $(x^2D^2 - xD - 3)y = x^2 \log x$, where $D \equiv \frac{d}{dx}$.

5 + 5 + 5

11. a) Prove that the set D of all odd integers forms a commutative group with respect to the composition $*$ defined by $a * b = a + b - 1 \forall a, b \in D$.

b) Solve the equation $x^3 - 7x^2 + 36 = 0$, given that one of its roots is double of another.

c) Find all complex numbers of the forms $z = a + bi$, where a and b are real numbers such that $zz' = 25$ and $a + b = 7$, where z' is the complex conjugate of z. 5 + 5 + 5