2014

Mathematics

Time Alloted : 3 Hours *Full Marks : 70*

The figure in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable

GROUP - A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following: 10x1=10

i) If a=(1,0,3) and b=(-1,2,5) then a+3b is equal to
   a) (-2,6,18) b) (2,-6,-18)
   c) (2,-6,18) d) (I,3,5)

ii) If $\sum_n |a_n|$ is convergent, then $\sum_n a_n$ is
   a) convergent b) divergent
   c) oscillatory d) none of these.

iii) A bounded sequence is
   a) Convergent b) divergent
   c) Oscillatory d) none of these

iv) The series $\sum \frac{1}{n\sqrt{n+1}}$ is
   a) convergent b) divergent
   c) oscillatory d)none of these

v) The integrating factor of $\frac{dy}{dx} + 2xy = x^3$ is

a) x^3 b) x^2

c) e^{x^2} d) e^{x^3}

vi) The infinite series $\sum_{n=1}^{\infty} \frac{n}{n+1}$ is

a) Convergent b) Divergent

c) Oscillatory d) None of these

vii) If the vectors (5, 2, 3), (7, 3, a), (9, 4, 5) of a vector space R^3 over R be linearly independent, then the value of a is not equal to

a) 2 b) 3

c) 1 d) 0

viii) The sequence 1, $\frac{1}{2}$, $\frac{1}{3}$ ..... $\frac{1}{n}$ is converges to

a) ∞ b) 0

c) 1 d) $\frac{1}{2}$

ix) The order and degree of the differential equation

$\frac{d^2y}{dx^2} = 1 + 2\left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right)^3$ are

a) 2, 1 b) 1, 2

c) 1, 3 d) 3, 1

x) The sequence $\{(-1)^n\}$ is

a) Convergent b) Oscillatory

c) Divergent d) None of these

xi) The general solution of log $\frac{dy}{dx} = x - y$ is

a) $e^y - e^x = c$ b) $e^y + e^x = c$

c) $e^{y+x} = c$ d) $e^{x-y} = c$

xii) Which of the following pair can form a basis of R^2?

a) {(1,2),(2,4)} b) {(0,0),(3,33)}

c) {(2,2),(3,3)} d) {(1,1),(1,2)}

xiii) The particular integral of $(d^2y/dx)^2 - 3(dy/dx) + 2y = \sin 3x$ is

a) 1/130 (9cos3x - 7sin 3x)

b) 1/130 (7cos3x - 9sin3x)

c) 1/130 sin3x

d) none of these

GROUP - B

(Short Answer Type Questions)

Answer any *three* of the following. 3x5=15

2. Prove that the vectors {(1,2,2),(2,1,2),(2,2,1)} are linearly independent in R^3.
3. Test the convergence of the series: $1+\frac{2}{1!}+\frac{2^2}{2!}+\frac{2^3}{3!}+\frac{2^4}{4!}+.....$
4. Solve: $e^y\left(1+x^2\right)\frac{dy}{dx}-2x\left(1+e^y\right)=0$
5. Define a subspace of a vector space. Show that the intersection of two subspaces of a vector space is a subspace.
6. Show that the sequence $\sqrt{2},\sqrt{2+\sqrt{2}},\sqrt{2+\sqrt{2+\sqrt{2}}}.....$ Converges to 2.

GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. 3x15=45

7. a) Test the convergence of the following series: $\sum \frac{n^2-1}{n^2+1}x^n$

   b) Examine whether the differential equation

   $(e^y+1)\cos\ xdx+e^y\sin\ ydy=0$ is exact or not.

   c) Find the basis and the dimension of the subspace W of R^3 where $W=\{(x,y,z)\in R^3 : 2x-y+3z=0\}$

8. a) Solve $\frac{dy}{dx}=\sin(x+y)$

   b) Let $T:R^2\to R^2$ be a linear transformation such that T(1, 1) = (2, -3) and T(1, -1) = (4,7). Find the matrix of T.

   c) Prove that the sequence $\left\{\frac{1}{(n+1)^2}+\frac{1}{(n+2)^2}+\ldots\ldots\frac{1}{(2n)^2}\right\}$ is convergent. Find its limit.

9. a) Form a differential equation by eliminating A and B from the following:
   y=A cos x + B sin x

   b) Find whether the following vectors are linearly dependent or not {(1,2,3),(2,3,1),(3,2,1)}

   c) Discuss the convergence of the series $\sum_{n=1}^{\infty}\frac{\cos n\pi}{n^2+1}$.

10. a) Solve: $\frac{dy}{dx}+y\ \tan\ x=y^3\cos\ x$

    b) For what values of x the three vectors (1,1,2), (x,1,1), (1,2,1) are linearly independent.

    c) Solve: $y=px+\sqrt{1+p^2}$

11. a) Prove that the vectors (x_1,y_1) and (x_2,y_2) are linearly dependent, if and only if $x_1y_2-x_2y_1=0$

12. b) Test the convergence of the series $\sum\frac{x^n}{n\sqrt{n+1}}$

    c) Find the linear transformations T, where

    T:$R^3\to R^2$ such that T(1,0,0) = (1,2,) T (0,1,0)=(1,-1) and T(0,0,1)=(1,0).