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**Paper Code : BM-201**

## MATHEMATICS

*Time Allotted* : 3 Hours *Full Marks* : 70

*The figures in the margin indicate full marks.*

### GROUP – A
### ( Multiple Choice Type Questions )

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by

   $T(x, y) = (x + 2y, x - y, y)$. Then the image of $(2, 2)$ is

   a) $(2, 1, -1)$ b) $(6, 1, -1)$

   c) $(6, 0, 2)$ d) none of these.

   ii) An integrating factor of $x\frac{dy}{dx} - y = 1$ is

   a) $x$ b) $\frac{1}{x}$

   c) $-x$ d) $-\frac{1}{x}$.

iii) If for a sequence $\{ u_n \}$, $\lim_{n \to \infty} u_n = 0$ then

a) $\{ u_n \}$ converges to 1

b) $\{ u_n \}$ converges to 0

c) $\{ u_n \}$ is divergent

d) none of these.

iv) The order and degree of the differential equation

$$\frac{d^2y}{dx^2} = \sqrt{\left(\frac{dy}{dx}\right)^3 - 2\frac{dy}{dx}}$$ are

a) ( 1, 3 ) b) ( 2, 3 )

c) ( 2, 2 ) d) ( 1, 2 ).

v) $\alpha$ be linear combination of the vectors $\beta$ and $\gamma$ in a vector space $V$ over a field $F$. Then the set

$S \equiv \{ \alpha, \beta, \gamma \}$ is

a) linearly independent

b) linearly dependent

c) $S$ forms a basis of $V$

d) none of these.

vi) If ( 2, 1 ) = $x$ ( 1, 2 ) + $y$ ( 0, 3 ) then the values of $x$ and $y$ are respectively

a) 3, 1 b) 2, – 1

c) 2, 0 d) 1, – 1.

vii) The value of $k$ for which the vectors ( 1, 2, 1 ), ( $k$, 1, 1 ) and ( 1, 1, 2 ) in $\mathbb{R}^3$ are linearly independent is

a) $k \neq -\frac{2}{3}$  b) $k \neq \frac{2}{3}$

c) $k \neq -\frac{3}{2}$  d) none of these.

viii) The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + \sqrt{n+1}}$ is

a) convergent  b) divergent

c) oscillatory  d) none of these.

ix) The order of the differential equation whose general solution is $y = a(x - a)^2$, where $a$ is an arbitrary constant is

a) 1  b) 2

c) 3  d) none of these.

x) $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ is convergent if

a) sequence $\{ a_n \}$ is monotonic decreasing

b) $\lim_{n \to \infty} a_n = 0$

c) both (a) and (b)

d) none of these.

xi) The vectors ( 1, 0, 0 ), ( 1, 1, 0 ) and ( 1, 1, 1 ) are

a) linearly dependent

b) linearly independent

c) a generating set of $\mathbb{R}^3$

d) none of these.

xii) In a linear mapping $T : V \to W$, which of the following is true ?

a) $Ker(T)$ may not be a vector space

b) $Ker(T)$ is a subset of $W$

c) $\theta \in Ker(T)$ where $\theta$ being the null vector in $V$

d) $\theta' \in Ker(T)$ where $\theta'$ being the null vector in $W$.

xiii) A differential equation $M\,dx + N\,dy = 0$ is exact when

a) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$

b) $\frac{\partial M}{\partial x} \neq \frac{\partial N}{\partial y}$

c) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

d) $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$.

## GROUP – B

**( Short Answer Type Questions )**

Answer any *three* of the following. $3 \times 5 = 15$

2. Find whether the following set of vectors is linearly independent or not :

$$\{(1, 2, 3), (2, 3, 1), (3, 2, 1)\}$$

3. Solve : $\dfrac{dy}{dx} = \dfrac{y}{x} + \tan\dfrac{y}{x}$.

4. Solve : $(D^2 - 2D + 1)\,y = xe^{2x}$, $D \equiv \dfrac{d}{dx}$.

5. State D'Alembert's ratio test. Test the convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$.

6. Find the representative matrix of the linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$T(x, y, z) = (x - 2y,\ y - 2z,\ z - 2x).$$

## GROUP – C

**( Long Answer Type Questions )**

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Let $S = \{(x, y, z) \in \mathbb{R}^3 \mid x + y + z = 0,\ x, y, z \in \mathbb{R}\}$. Prove that $S$ is a subspace of $\mathbb{R}^3$.

b) Discuss the convergence of the series

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$$

c) Find the basis of $M_2$ ( the family of all real square matrices of order 2 ). $5 + 5 + 5$

8. a) Show that the set $\{(2, 1, 1), (1, 2, 1), (1, 1, 2)\}$ is a basis of $\mathbb{R}^3$.

b) Solve : $(xy^2 - e^{1/x^3})\,dx - x^2y\,dy = 0$.

c) Solve : $y = px + \sqrt{a^2p^2 + b^2}$, where $p \equiv \dfrac{dy}{dx}$.

5 + 5 + 5

9. a) Examine the convergence of the following series for different values of $x$ :

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^2+1}}\, x^n$$

b) Test the convergence of the following series :

$$1 + \frac{1}{2} \cdot \frac{x^2}{4} + \frac{1.3.5}{2.4.6} \cdot \frac{x^4}{8} + \frac{1.3.5.7.9}{2.4.6.8.10} \cdot \frac{x^6}{12} + \ldots \infty .$$

c) Find a basis and the dimension of the subspace $W$ of $\mathbb{R}^3$, where

$$W = \{(x, y, z) \in \mathbb{R}^3 : x + 2y + z = 0,\ 2x + y + 3z = 0\}.$$

5 + 5 + 5

10. a) Determine the linear transform $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ which maps the basis vectors $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ of $\mathbb{R}^3$ to the vectors $(1, 1)$, $(2, 3)$, $(3, 2)$ respectively. Also find $Ker(T)$ and $Im(T)$.

b) Solve : $\dfrac{d^2y}{dx^2} + 9y = \cos 3x$.

c) Solve : $(x^2D^2 - xD - 3)\,y = x^2 \log x,\ D \equiv \dfrac{d}{dx}$.

5 + 5 + 5

11. a) Solve : $y = 2x \frac{dy}{dx} - \left( \frac{dy}{dx} \right)^2$.

b) Find the coordinate vector of ( 0, 3, 1 ) $\in \mathbb{R}^3$ relative to the basis { ( 1, 1, 0 ), ( 1, 0, 1 ), ( 0, 1, 1 ) }.

c) Discuss the convergence of the sequence $\left\{ \frac{n^2}{3^n} \right\}$.

OR

c) Find the differential equation of all circles touching the axis of $x$ at the origin. 5 + 5 + 5