MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Utech

MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

Paper Code : BM-201

MATHEMATICS

Time Allotted: 3 Hours | *Full Marks*: 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for *any ten* of the following: $1\times10=10$

(i) If $(3,1) = x(1,2) + y(0,3)$, then the values of x and y are respectively

(a) $3,-5$ (b) $3,1$
(c) $3,-\frac{5}{3}$ (d) $3,-\frac{5}{2}$

(ii) The equation of the curve which satisfies the differential equation $\frac{dy}{dx} = 0$ is

(a) a straight line passing through the origin (b) a straight line parallel to x-axis
(c) a straight line parallel to y-axis (d) None of these

(iii) $\{x^n\}$ converges if

(a) $x = 1$ (b) $x > 1$
(c) $x < -1$ (d) $-1 < x < 1$

(iv) The order and degree of the differential equation $\frac{d}{dx}\left(y.\frac{dy}{dx}\right) = 5$ are

(a) 1, 2 (b) 2, 2
(c) 1, 1 (d) 2, 1

(v) If $T(x,y,z) = (x,y,0)$ for all $(x,y,z) \in R^3$ is a linear transformation, then Kernel(T) is

(a) $\{(0,0,0)\}$ (b) x-axis
(c) y-axis (d) z-axis

(vi) The series $\sum \frac{1}{n^P}$ is convergent if

(a) $P \geq 1$ (b) $P > 1$

(c) $P < 1$ (d) $P \leq 1$

(vii) The sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$ is

(a) divergent (b) unbounded

(c) bounded (d) None of these

(viii) $\frac{1}{D^2}(x^4) = ?$

(a) x^5 (b) x^6

(c) $\frac{x^6}{30}$ (d) $\frac{x^4}{30}$

(ix) The I.F. of $\frac{dx}{dy} = x + y^2$ is

(a) e^x (b) e

(c) e^{2x} (d) e^{-x}

(x) T is a linear transformation by $T(x_1, x_2) = (-x_1, -x_2)$ then the dimension of $T(V_2)$ is

(a) 2 (b) 1

(c) 0 (d) None of these

(xi) A bounded monotonic increasing sequence converges to its

(a) any upper bound (b) greatest lower bound

(c) least upper bound (d) 0

(xii) The differential equation $(x\, e^{axy} + 2y)\frac{dy}{dx} + y.e^{xy} = 0$ is exact for $a = ?$

(a) 3 (b) 1

(c) 2 (d) 0

Group – B

Answer *any three* questions: **5x3=15**

2. Show that the differential equation of all parabolas with foci at the origin and axes along the x-axis is

$$y\left(\frac{dy}{dx}\right)^2 + 2x\frac{dy}{dx} - y = 0$$

3. Find the value of x such that the vectors $(1, 2, 1)$, $(x, 3, 1)$ and $(2, x, 0)$ are linearly dependent.

4. Examine the convergence of the sequence $\left\{\frac{n^n}{\underline{|n}}\right\}$.

5. Solve : $(D^2 - 4)y = e^{2x} + e^{-4x}$

6. Show that $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots\ldots$ to ∞ is a divergent series. (15)

Group – C

Answer *any three* questions: 15×3=45

7. (a) Solve : $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$ 5

(b) Let $W = \{(x, y, z) \in \mathbb{R}^3 : x + y - 2z = 0\}$. Show that W is a subspace of $\mathbb{R}^3$. Find a basis and dimension of W. 3+1+1=5

(c) From the definition, prove that, $\lim_{n\to\infty} x^n = 0$, when $-1 < x < 1$. 5

8. (a) Prove that the vectors $(1, -2, 3), (2, 3, 1)$ and $(-1, 3, 2)$ form a basis of V_3.

(b) Solve : $(x + y + 1)\frac{dy}{dx} = 1$

(c) Discuss the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$

9. (a) Solve : $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = x^2 e^{3x}$ 5

(b) Determine the linear mapping $T: \mathbb{R}^3 \to \mathbb{R}^3$ which maps the basis vectors $(0, 1, 1), (1, 0, 1), (1, 1, 0)$ of $\mathbb{R}^3$ to the vectors $(1, 2, 1), (1, 1, 2), (2, 1, 1)$ respectively. Find dim(ker(T)). 5+2=7

(c) Obtain a singular solution of the equation: $(y - px)(p - 1) = p; p = \frac{dy}{dx}$

10. (a) If $T: \mathbb{R}^3 \to \mathbb{R}^3$ is given by $T(x, y, z) = (x - y, y - z, z - x)$; show that T is a linear transformation and obtain ker(T).

(b) Solve : $(D^2 + 1)y = \sin 2x$

(c) State Leibnitz's test. Using this show that $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots$ to ∞ is convergent. (5)

11. (a) Solve : $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = \log x$ 5 (15)

(b) Solve : $(D^2 + 4)y = x \sin x$

(c) Test the convergence of the series $\frac{6}{1.3.5} + \frac{8}{3.5.7} + \frac{10}{5.7.9} + \ldots\ldots$ to ∞.