MATHEMATICS (SEMESTER - 2)

CS/BCA/SEM-2/BM-201/09

1. ....
Signature of Invigilator

2. .... **Reg. No.**
Signature of the Officer-in-Charge

Roll No. of the Candidate

CS/BCA/SEM-2/BM-201/09

ENGINEERING & MANAGEMENT EXAMINATIONS, JUNE – 2009

MATHEMATICS (SEMESTER - 2)

Time : 3 Hours] [Full Marks : 70

INSTRUCTIONS TO THE CANDIDATES :

1. This Booklet is a Question-cum-Answer Booklet. The Booklet consists of **32 pages**. The questions of this concerned subject commence from Page No. 3.
2. a) In **Group – A**, Questions are of Multiple Choice type. You have to write the correct choice in the box provided **against each question**.
   b) For **Groups – B** & **C** you have to answer the questions in the space provided marked 'Answer Sheet'. Questions of **Group – B** are Short answer type. Questions of **Group – C** are Long answer type. Write on both sides of the paper.
3. **Fill in your Roll No. in the box** provided as in your Admit Card before answering the questions.
4. Read the instructions given inside carefully before answering.
5. You should not forget to write the corresponding question numbers while answering.
6. Do not write your name or put any special mark in the booklet that may disclose your identity, which will render you liable to disqualification. Any candidate found copying will be subject to Disciplinary Action under the relevant rules.
7. **Use of Mobile Phone and Programmable Calculator is totally prohibited in the examination hall.**
8. You should return the booklet to the invigilator at the end of the examination and should not take any page of this booklet with you outside the examination hall, **which will lead to disqualification**.
9. Rough work, if necessary is to be done in this booklet only and cross it through.

No additional sheets are to be used and no loose paper will be provided

FOR OFFICE USE / EVALUATION ONLY

Marks Obtained

Group – A Group – B Group – C

Question Number																*Total Marks*	*Examiner's Signature*
Marks Obtained																	

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Head-Examiner/Co-Ordinator/Scrutineer

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MATHEMATICS

SEMESTER - 2

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GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) If $\alpha = (1, 0, 3)$ and $\beta = (-1, 2, 5)$, then $\alpha + 3\beta$ is equal to

a) $(-2, 6, 18)$ b) $(2, -6, -18)$

c) $(2, -6, 18)$ d) $(-1, -3, 5)$.

ii) The basis of a vector space contains

a) linearly independent set of vectors

b) linearly dependent set of vectors

c) scalars only

d) none of these.

iii) Integrating factor of $x\,dx = -y\,dy$ is

a) $1/(xy)$ b) $1/(x^2 + y^2)$

c) $1/y^2$ d) none of these.

iv) The infinite series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if

a) $p = 1$ b) $p > 1$

c) $p < 1$ d) none of these.

v) The order and degree of the differential equation $\left(\frac{d^2 y}{dx^2} \right)^{2/3} - 3 \frac{dy}{dx} = 4$ are

a) 2, 2 b) $2, \frac{2}{3}$

c) 2, 1 d) 2, 3.

vi) If the three vectors (5, 2, 3), (7, 3, x) and (9, 4, 5) are linearly dependent, then x is

a) 1 b) 2

c) 3 d) 4.

vii) If $\lim_{n \to \infty} a_n = 0$, then the series $\Sigma (-1)^n a_n$ is

a) convergent b) divergent

c) oscillatory d) none of these.

viii) The family of curves $y = e^x (A \cos x + B \sin x)$ is represented by the differential equation

a) $\frac{d^2 y}{dx^2} = 2 \frac{dy}{dx} - y$ b) $\frac{d^2 y}{dx^2} = 2 \frac{dy}{dx} - 2y$

c) $\frac{d^2 y}{dx^2} = \frac{dy}{dx} - 2y$ d) $\frac{d^2 y}{dx^2} = 2 \frac{dy}{dx} + y$.

ix) The sequence $1, \frac{1}{2}, \frac{1}{3}, \ldots \frac{1}{n}, \ldots\ldots$ converges to

a) • b) 0

c) 1 d) $\frac{1}{2}$.

x) The four vectors (1, 1, 0, 0), (1, 0, 0, 1), (1, 0, a, 0,) and (0, 1, a, b) are linearly independent if

a) $a \pi 0, b \pi 2$ b) $a \pi 2, b \pi 0$

c) $a \pi 0, b \pi - 2$ d) $a \pi - 2, b \pi 0$.

xi) The solution of $\log\left(\frac{dy}{dx}\right) = ax + by$ is

a) $b\, e^{-ax} + a\, e^{-by} + k = 0$ b) $b\, e^{ax} + a\, e^{-by} + k = 0$

c) $b\, e^{-ax} + a\, e^{by} + k = 0$ d) $b\, e^{ax} + a\, e^{by} + k = 0$.

xii) $\lim\limits_{n \varnothing \bullet} \frac{x^n}{\lfloor n}$ is equal to

a) 0 b) 1

c) – 1 d) none of these.

xiii) The lower bound of the sequence $\left\{ \frac{(-1)^{n-1}}{n!} \right\}$) is

a) $-\frac{1}{2}$ b) $\frac{1}{2}$

c) 1 d) 0.

xiv) The value of $\lim\limits_{n \varnothing \bullet} \log(1/n)$ is equal to

a) 0 b) 1

c) – • d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Find the equation of curve whose slope at any point (x, y) on it is $2y$ and which passes through the point (3, 1).

3. Test for the convergence of the series :

$$x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \ldots\ldots, \; x > 0.$$

4. Examine whether the vectors (1, 2, 3, 0), (2, 1, 0, 3), (1, 1, 1, 1) and (2, 3, 4, 1) are linearly dependent or not. If yes, find among them which are independent.

5. Solve any three :

   a) $x \dfrac{dy}{dx} + y = y^2 \log x$

   b) $\cos^2 x \dfrac{dy}{dx} + y = \tan x$

   c) $y = px - \dfrac{a}{p}$ where $p = \dfrac{dy}{dx}$

   d) $\left(D^2 - 2D + 1 \right) y = x e^x$ where $D = \dfrac{d}{dx}$.

6. Define the limit of a sequence. Find

$$\lim_{n \to \infty} \left[\frac{1}{n^2} + \frac{2}{n^2} + \ldots + \frac{n}{n^2} \right] .$$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Define basis of a vector space V. Show that $\alpha_1 = (1, 0, 0)$, $\alpha_2 = (0, 1, 0)$ and $\alpha_3 = (0, 0, 1)$ form a basis of the vector space V_3. 5

   b) If $\{ \alpha, \beta, \gamma \}$ be a basis of real vector space V and $c \neq 0$ be a real number, examine whether $\{ \alpha + c\beta, \beta + c\gamma, \gamma + c\alpha \}$ is a basis of V or not. 5

   c) Find the value of k for which the vectors (1, 2, 1), (k, 1, 1) and (1, 1, 2) in R^3 are linearly dependent. 5

8. Test the convergence of any *three* of the following series : $3 \times 5 = 15$

a) $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^2}{10} + \ldots\ldots$

b) $\left(\frac{1}{3}\right)^2 + \left(\frac{1.2}{3.5}\right)^2 + \left(\frac{1.2.3}{3.5.7}\right)^2 + \ldots\ldots$

c) $\sum_{n=1}^{\infty} \left(1 + \frac{1}{\sqrt{n}}\right)^{-\sqrt{n}}$

d) $\sum_{n=1}^{\infty} \left(\frac{\cos nx}{n^2}\right)$.

9. Solve any *three* of the following : $3 \times 5 = 15$

a) $\frac{dy}{dx} + \frac{y \log y}{x} = \frac{y (\log y)^2}{x^2}$

b) $y = 2px - p^2$ where $p = \frac{dy}{dx}$

c) $e^x \sin y \, dx + (e^x + 1) \cos y \, dy = 0$

d) $(D^2 - 2D) y = e^x \sin x$.

10. a) Prove that $s = \{(0, 1, 1), (1, 0, 1), (1, 1, 0)$ is a basis of R^3 .

b) Show that $w = \{(x, y, z) \in R^3 / x + y + z = 0\}$ is a sub-space of R^3 and find a basis of w.

c) Determine K so that the set S is linearly dependent in R^3

$$S = \{(1, 2, 1), (k, 3, 1), (2, k, 0)\}.$$

5 + 5 + 5

11. a) Define the linear sum of two sets of vectors S and T.

b) If S and T are two sub-spaces of a vector space V, obtain a relation between rank (S), rank (T) and rank (V).

c) Let $T : R^2 \to R^2$ be a linear transformation such that $T(1, 1) = (2, -3)$ and $T(1, -1) = (4, 7)$. Find the matrix of T. 3 + 6 + 6

12. a) Using D' Alembert's ratio test, show that the following series is convergent :

$$x^2 + (2^2 / 3.4)\,x^4 + (2^2.4^2 / 3.4.5.6)\,x^6 + \ldots \infty$$

b) Prove that every absolutely convergent series is convergent.

c) Show that the following series is convergent :

$$u_n = \sqrt{n^3 + 1} - \sqrt{n^3} \text{ in } n \in [1, \infty].$$

END