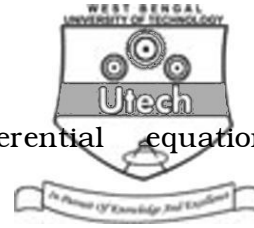


CS/BCA/SEM-2/BM-201/2010



xii) Auxiliary equation of the differential equation $\frac{d^2y}{dx^2} + 4y = \sin x$ is

a) $y = \cos 2x + \sin 2x$

b) $y = c_1 \cos 2x + c_2 \sin 2x$

c) $y = c_1 \cos x + \sin 2x$

d) none of these.

xiii) The general solution of $\log \frac{dy}{dx} = x - y$ is

a) $e^y - e^x = c$

b) $e^x + e^y = c$

c) $e^{x+y} = c$

d) $e^{x-y} = c.$

xiv) If S and T be two subspaces of a vector space V , then which of the following is also a subspace of V ?

a) $S \cup T$

b) $S - T$

c) $T - S$

d) $S \cap T.$

GROUP - B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Show that the sequence $\{2 + (-1)^n / n\}$ is convergent.

3. Solve : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2 + e^{3x}$



4. Find the value of x for which the vectors $(1, 2, 1)$, $(x, 3, 1)$ and $(2, x, 0)$ become linearly independent.
5. Find the value of the limit $\lim_{n \rightarrow \infty} (4n^3 + 6n - 7)/(n^3 - 2n^2 + 1)$.
6. Find a basis and the dimension of $S \cap T$, where S and T are subspaces of R^3 defined by

$$S = \{ (x, y, z) \in R^3 : 2x + y + 3z = 0 \}$$

$$\text{and } T = \{ (x, y, z) \in R^3 : x + 2y + z = 0 \}$$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Show that $\left\{ \frac{1}{\sqrt{n^2 + 1}} + \frac{1}{\sqrt{n^2 + 2}} + \dots + \frac{1}{\sqrt{n^2 + n}} \right\}$ is convergent and converges to 1.
- b) Show that the sequence $\sqrt{2}, \sqrt{2 + \sqrt{2}}, \sqrt{2 + \sqrt{2 + \sqrt{2}}}, \dots$ converges to 2. 8 + 7
8. Solve the following equations : 3 × 5
- a) $(D^2 - 2D + 1)y = x \sin x$
- b) $\frac{d^2y}{dx^2} + \frac{1}{x} \cdot \frac{dy}{dx} = \frac{12 \log x}{x^2}$



c) $3 \frac{dy}{dx} + 2 \frac{y}{x+1} = \frac{x^3}{y^2}$

9. a) Prove that a subset S of a vector space V over \mathbb{R} is a subspace if and only if $\alpha x + \beta y \in S$ for all $\alpha, \beta \in \mathbb{R}$ and $x, y \in S$.

b) Prove that the vectors $\{ (1, 2, 2), (2, 1, 2), (2, 2, 1) \}$ are linearly independent in \mathbb{R}^3 .

c) Find the basis and the dimension of the subspace W of \mathbb{R}^3 where

$$W = \{ (x, y, z) \in \mathbb{R}^3 : x + y + z = 0 \} \qquad 5 + 5 + 5$$

10. a) Solve $(px - y)(py + x) = a^2p$, by using the substitution $x^2 = u, y^2 = v$; where $p = \frac{dy}{dx}$.

b) Obtain the general solution and singular solution of the equation $y = px + \sqrt{a^2p^2 + b^2}$. 7 + 8

11. a) Define basis of a vector space.

b) Show that the vectors $\alpha_1 = (1, 0, -1), \alpha_2 = (1, 2, 1)$ and $\alpha_3 = (0, -3, 2)$ form a basis for \mathbb{R}^3 . Express $(1, 0, 0)$ as a linear combination of α_1, α_2 and α_3 .



c) Find the matrix of the linear transformation T on $V_3(\mathbb{R})$ defined as

$T(a, b, c) = (2b + c, a - 4b, 3a)$ with respect to the ordered basis B where

$$B = \{ (1, 1, 1), (1, 1, 0), (1, 0, 0) \}. \quad 3 + 6 + 6$$

12. a) Prove that the sequence $\{a_n\}$ is monotonically increasing and bounded when

$$a_n = (3n + 1)/(n + 2)$$

b) State D' Alembert's Ratio Test.

c) If α, β, γ form a basis of a vector space V , then prove that $\alpha + \gamma, 2\alpha + 3\beta + 4\gamma$ and $\alpha + 2\beta + 3\gamma$ also form a basis of the vector space V . 8 + 2 + 5

