Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/BCA/SEM-2/BM-201/2010

2010

MATHEMATICS

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10

   i) The basis of a vector space contains

      a) linearly independent set of vectors

      b) linearly dependent set of vectors

      c) scalars only

      d) none of these.

   ii) The solution of $\frac{d^2y}{dx^2} = 0$ is

      a) $y = e^x$ b) $y = 0$

      c) $y = \sin x$ d) $y = \log_e x$.

iii) If $f(3, 1) = x(1, 2) + y(0, 3)$ then the values of x and y are respectively

a) $(3, -5)$ b) $(3, 1)$

c) $(3, -5/3)$ d) $(3, -5/2)$.

iv) $\lim_{n \to \infty} (3n + 1)/(2n - 3)$ is

a) $\frac{1}{2}$ b) $\frac{3}{2}$

c) 1 d) $-\frac{1}{3}$.

v) The value of $\left(1/D^2\right)\left(x^3\right)$ is

a) x^5 b) $\frac{1}{20}$

c) 20 d) $\frac{1}{20}x^5$.

vi) $\sum 1/n^p$ is divergent if

a) $p \leq 1$ b) $p > 1$

c) $p < 1$ d) $p = 1$.

vii) If $P = \{2, 4, 6, 7, 8, 9\}$, $Q = \{1, 2, 6, 9\}$, then $P - Q$ is

a) $\{4, 7, 8\}$

b) $\{4, 6, 8, 9\}$

c) $\{1\}$

d) $\{2, 4, 6, 7, 8, 9\}$.

viii) $\frac{1}{(D-2)(D-3)}e^{2x}$ is

a) $-e^{2x}$ b) xe^{2x}

c) $-xe^{2x}$ d) $-xe^{3x}$.

ix) Integrating factor of $\frac{dy}{dx}+y=x$ is

a) e^{-x} b) e^{x}

c) x^2 d) none of these.

x) The differential equation $\left(\frac{dy}{dx}\right)^2+ay^{\frac{1}{2}}=x$ is

a) linear of degree 2

b) non-linear of order one and degree 4

c) non-linear of order one and degree 2

d) none of these.

xi) If vectors (a, 0, 1), (0, 1, 0), (1, a, 1) of a vector space $\mathbb{R}^3$ over $\mathbb{R}$ be linearly dependent, then the value of a is

a) 2

b) 3

c) 1

d) none of these.

xii) Auxiliary equation of the differential equation $\frac{d^2y}{dx^2}+4y=\sin x$ is

a) $y = \cos 2x + \sin 2x$

b) $y=c_1 \cos 2x+c_2 \sin 2x$

c) $y=c_1 \cos x+\sin 2x$

d) none of these.

xiii) The general solution of $\log \frac{dy}{dx}=x-y$ is

a) $e^y-e^x=c$

b) $e^x+e^y=c$

c) $e^{x+y}=c$

d) $e^{x-y}=c$.

xiv) If S and T be two subspaces of a vector space V, then which of the following is also a subspace of V ?

a) $S \cup T$

b) $S - T$

c) $T - S$

d) $S \cap T$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Show that the sequence $\{2 + (-1)^n \ 1/n\}$ is convergent.

3. Solve : $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x^2+e^{3x}$

4. Find the value of x for which the vectors (1, 2, 1), (x, 3, 1) and (2, x, 0) become linearly independent.

5. Find the value of the limit $\lim_{n \to \infty} (4n^3 + 6n - 7)/(n^3 - 2n^2 + 1)$.

6. Find a basis and the dimension of $S \cap T$, where S and T are subspaces of R^3 defined by

$$S = \{ (x, y, z) \in R^3 : 2x + y + 3z = 0 \}$$

and $T = \{ (x, y, z) \in R^3 : x + 2y + z = 0 \}$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

7. a) Show that $\left\{ \frac{1}{\sqrt{n^2 + 1}} + \frac{1}{\sqrt{n^2 + 2}} + \dots + \frac{1}{\sqrt{n^2 + n}} \right\}$ is convergent and converges to 1.

b) Show that the sequence $\sqrt{2}, \sqrt{2 + \sqrt{2}}, \sqrt{2 + \sqrt{2 + \sqrt{2}}}, \dots$ converges to 2. 8 + 7

8. Solve the following equations : 3 × 5

a) $(D^2 - 2D + 1)y = x \sin x$

b) $\frac{d^2y}{dx^2} + \frac{1}{x} \cdot \frac{dy}{dx} = \frac{12 \log x}{x^2}$

c) $3\frac{dy}{dx} + 2\frac{y}{x+1} = \frac{x^3}{y^2}$

9. a) Prove that a subset S of a vector space V over $\mathbb{R}$ is a subspace if and only if $\alpha x + \beta y \in S$ for all $\alpha, \beta \in \mathbb{R}$ and $x, y \in S$.

b) Prove that the vectors $\{ (1, 2, 2), (2, 1, 2), (2, 2, 1) \}$ are linearly independent in $\mathbb{R}^3$.

c) Find the basis and the dimension of the subspace W of $\mathbb{R}^3$ where

$W = \{ (x, y, z) \in \mathbb{R}^3 : x + y + z = 0 \}$ 5 + 5 + 5

10. a) Solve $(px - y)(py + x) = a^2 p$, by using the substitution $x^2 = u$, $y^2 = v$; where $p = \frac{dy}{dx}$.

b) Obtain the general solution and singular solution of the equation $y = px + \sqrt{a^2 p^2 + b^2}$. 7 + 8

11. a) Define basis of a vector space.

b) Show that the vectors $\alpha_1 = (1, 0, -1), \alpha_2 = (1, 2, 1)$ and $\alpha_3 = (0, -3, 2)$ form a basis for $\mathbb{R}^3$. Express $(1, 0, 0)$ as a linear combination of α_1, α_2 and α_3.

c) Find the matrix of the linear transformation T on $V_3(\mathbb{R})$ defined as

$T(a, b, c) = (2b + c, a - 4b, 3a)$ with respect to the ordered basis B where

$B = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$. 3 + 6 + 6

12. a) Prove that the sequence $\{a_n\}$ is monotonically increasing and bounded when

$a_n = (3n+1)/(n+2)$

b) State D' Alembert's Ratio Test.

c) If α, β, γ form a basis of a vector space V, then prove that $\alpha + \gamma$, $2\alpha + 3\beta + 4\gamma$ and $\alpha + 2\beta + 3\gamma$ also form a basis of the vector space V. 8 + 2 + 5