

Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS / BCA / SEM-2 / BM-201 / 2011**

**2011**

**MATHEMATICS**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.  
Candidates are required to give their answers in their own words  
as far as practicable.*

**Group - A**

**( Multiple Choice Type Questions )**

1. Choose the correct alternatives for any *ten* of the following :

10 × 1 = 10

i) The degree and order of the differential equation

$$y = \frac{x d^2 y}{dx^2} + r \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \text{ are}$$

- a) 2, 2
- b) 2, 1
- c) 3, 2
- d) none of these.

ii) The geometric series  $1+r+r^2+\dots$  is convergent if

- a)  $-1 < r < 1$
- b)  $r > 1$
- c)  $r = 1$
- d) none of these.

iii) The series  $1 + 1 + 1 + \dots$  is

- a) convergent
- b) divergent
- c) oscillatory
- d) none of these.



- iv) An absolutely convergent series is
- a) necessarily convergent
  - b) not necessarily convergent
  - c) conditionally convergent
  - d) none of these.
- v) Leibnitz's test is applied to
- a) a constant series
  - b) an alternating series
  - c) series of positive terms only
  - d) none of these.
- vi) If  $W_1$  and  $W_2$  be two subspaces of a vector space  $V(F)$  then  $W_1 \cap W_2$  is
- a) necessarily a subspace
  - b) not a subspace
  - c) is a subspace only when one is contained within another
  - d) none of these.
- vii) In the vector space  $R^3$  over the field  $R$  the vectors  $(1, 0, 0)$ ,  $(0, 1, 0)$  and  $(0, 0, 1)$  are
- a) linearly independent
  - b) linearly dependent
  - c) none of these.





xiii) If  $T : V_2 \rightarrow V_3$  be defined by

$T(v_2) = \{ (x, 0, 0) : x \text{ is a real number} \}$  then rank of  $T$  is

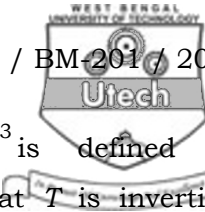
- a) 3
  - b) 2
  - c) 1
  - d) 0.
- xiv) If  $T$  is a linear transformation from vector space  $V$  into  $W$ , then
- a)  $\text{rank}(T) + \text{nullity}(T) = \dim(V)$
  - b)  $\text{rank}(T) + \text{nullity}(T) = \dim(W)$
  - c)  $\text{rank}(T) - \text{nullity}(T) = \dim(V)$
  - d) none of these.

**Group – B**

**( Short Answer Type Questions )**

Answer any *three* of the following.  $3 \times 5 = 15$

- 2. Test the convergence of the series  $\sum_{n=1}^{\infty} \frac{n!}{n}$
- 3. Show that the mapping defined by  $T : R^2 \rightarrow R^3$   
 $T(x_1, x_2) = (x_1 + x_2, x_1 - x_2, x_2)$  is linear. Find the value of  $T(1, 2)$ .
- 4. Solve :  
$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 3y = x^2 \log x .$$



5. The linear transformation  $T: R^3 \rightarrow R^3$  is defined by  $T(x, y, z) = (x - y, x + 2y, y + 3z)$ . Show that  $T$  is invertible and determine  $T^{-1}$ .
6. Prove that the set of vectors  $\{ (1, -2, 3), (2, 3, 1), (-1, 3, 2) \}$  is linearly independent. Also verify whether this set forms a basis of  $V_3$  or not.
7. Let  $S = \{ (x, y, z) : (x, y, z) \in R^3, x + y + z = 0 \}$ . Prove that  $S$  is a subspace. Find the dimension of  $S$ .

**Group - C**

**( Long Answer Type Questions )**

Answer any *three* of the following.  $3 \times 15 = 45$

8. a) Find the order and degree of the differential equation

$$\frac{d^2 y}{dx^2} = \left( y + \left( \frac{dy}{dx} \right)^2 \right)^{1/4}$$

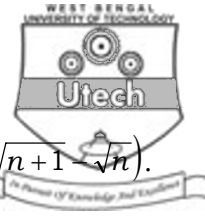
- b) Verify whether the differential equation  $e^y dx + (xe^y + 2y) dy = 0$  is exact. If so, then solve it.

- c) Solve the differential equation  $x \frac{dy}{dx} - 2y = xy^4$ .

3 + 5 + 7

9. a) Solve  $y = px + \frac{a}{p}$  and also obtain the singular solution.

- b) Solve  $x^2 \frac{d^2 y}{dx^2} - 3x \frac{dy}{dx} - 5y = \sin(\log x)$ .



- c) Test the convergence of the series  $\sum_{n=1}^{\infty} (\sqrt{n+1} - \sqrt{n})$ .  
 5 + 6 + 4

10. a) State D' Alembert's ratio test. Test the convergence of the series  $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$ .

- b) Prove that the series  $\left(1 + \frac{1}{2}\right) - \left(1 + \frac{1}{4}\right) + \left(1 + \frac{1}{8}\right) - \left(1 + \frac{1}{16}\right) + \dots$  is an oscillating series.

- c) Verify for the following example that the converse of the statement "If  $\sum a_n$  be a convergent series then

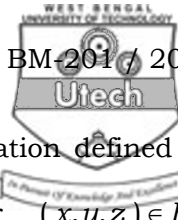
$$\lim_{n \rightarrow \infty} a_n = 0$$

is not true, where  $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{n}$ .

5 + 6 + 4

11. a) Define basis of a vector space.  
 b) Show that the vectors  $\alpha_1 = (1, 0, -1)$ ,  $\alpha_2 = (1, 2, 1)$  and  $\alpha_3 = (0, -3, 2)$  form a basis for  $R^3$ .

- c) Determine the value of  $k$  so that the set  $S = \{ (1, 2, 1), (k, 3, 1), (2, k, 0) \}$  is linearly dependent in  $R^3$ .  
 3 + 6 + 6



12. a) Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be a linear transformation defined by  $T(x, y, z) = (x - 2y, y - 2z, z - 2x)$  for  $(x, y, z) \in \mathbb{R}^3$ . Obtain a matrix representation for the linear transformation  $T$ .

b) Let  $V =$  set of all  $2 \times 2$  matrices and  $T : V \rightarrow V$  be defined by  $T(X) = AX - XA$  where  $X \in V$  and  $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ . Find the basis of  $\ker(T)$  and the nullity.

c) Let  $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$  be a linear mapping defined by  $T(x, y, z) = (x + 2y, z)$ . Verify that  $\text{dimension}(\ker(T)) + \text{dimension}(\text{Image}(T)) = \dim(\mathbb{R}^3)$ . 5 + 6 + 4

13. a) Define a subspace of a vector space.

b) State a necessary and sufficient condition for  $W \subseteq V$  to be a subspace of  $V(F)$ .

c) Test whether the series

$$\frac{x}{1+x} - \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} - \frac{x^4}{1+x^4} + \dots \quad (0 < x < 1)$$

is convergent or not. 3 + 3 + 9

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