



Name :
Roll No. :
Invigilator's Signature :

CS/BCA/SEM-2/BM-201/2012

**2012
MATHEMATICS**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words
as far as practicable.*

**GROUP - A
(Multiple Choice Type Questions)**

1. Choose the correct alternative for any *ten* the following:
10 × 1 = 10

i) Integrating factor of the differential equation

$$x(1-x^2) dy + (2x^2y - y - ax^3) dx = 0 \text{ is}$$

- a) $x/\sqrt{1-x^2}$ b) $x/\sqrt{x(x^2-1)}$
c) $x/\sqrt{x^2-1}$ d) $x^2/\sqrt{1-x^2}$.

ii) The order and degree of the differential equation

$$\sqrt{d^2y/dx^2} + dy/dx = y \text{ are}$$

- a) 2, 1 b) 1, 2
c) 2, 2 d) 2, 3.

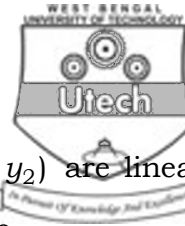


GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Solve : $\frac{d^2y}{dx^2} + \frac{1}{x} \frac{dy}{dx} = 12 \frac{\log x}{x^2}$
- b) Obtain the general solution and singular solution of the equation $y = px + \sqrt{a^2 p^2 + b^2}$
- c) Solve : $3 \frac{dy}{dx} + \frac{2y}{x+1} = \frac{x^3}{y^2}$ 5 + 6 + 4
9. a) State Leibnitz theorem for Alternating series and test the convergence of the series $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$
- b) Test the convergence of the following series $\frac{1}{2} + \frac{1}{3} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^3} + \dots$
- c) Show that the sequence $\left\{ 2 + \frac{(-1)^n}{n} \right\}$ is convergent. 6 + 5 + 4
10. a) Solve $\frac{d^2y}{dx^2} + 4 \frac{dy}{dx} + 29 y = 0$
when $x = 0$, $y = 0$, $\frac{dy}{dx} = 15$
- b) Show that the sequence $\sqrt{2} , \sqrt{2 + \sqrt{2}} , \sqrt{2 + \sqrt{2 + \sqrt{2}}} , \dots$ converges to 2.
- c) Define basis and dimension of a vector space. 6 + 6 + 3



11. a) Prove that the vectors (x_1, y_1) and (x_2, y_2) are linearly dependent, if and only if $x_1 y_2 - x_2 y_1 = 0$.
- b) Show that the vectors $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 2, 1)$ and $\alpha_3 = (0, -3, 2)$ form a basis of R^3 . Express $(1, 0, 0)$ as a linear combination of α_1, α_2 and α_3 .
- c) If $\alpha_1, \alpha_2, \alpha_3$ form a basis of a vector space V , then prove that $\alpha_1 + \alpha_3, 2\alpha_1 + 3\alpha_2 + 4\alpha_3$ and $\alpha_1 + 2\alpha_2 + 3\alpha_3$ also form a basis of the vector space V . 4 + 6 + 5
12. a) Let T be defined by $T(x, y) = (x', y')$ where $x' = x \cos \theta - y \sin \theta, y' = x \sin \theta + y \cos \theta$
 Prove that T is a linear transformation.
- b) The linear transformation $T : R^3 \rightarrow R^3$ transforms the basis vectors $(1, 2, 1), (2, 1, 0)$ & $(1, -1, -2)$ to the basis vectors $(1, 0, 0), (0, 1, 0)$ & $(0, 0, 1)$ respectively. Find T . Hence find $T(3, -3, 3)$.
- c) Find the Kernel, Image, Nullity and Rank of $T : R^3 \rightarrow R^2$ where
 $T(1, 0, 0) = (2, 1)$
 $T(0, 1, 0) = (0, 1)$
 $T(0, 0, 1) = (1, 1)$ 4 + 7 + 4



13. a) Prove that a subset S of a vector space V over R is a subspace if and only if $\alpha x + \beta y \in S$ for all $\alpha, \beta \in R$ and $x, y \in S$.
- b) Show that the family M_2 of all real square matrices of order 2 forms a vector space over reals, and find a basis for M_2 .
- c) Let $S = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid a + b = 0 \text{ and } a, b, c, d \in R \right\}$

Prove that S is a subspace of M_2 .

5 + 6 + 4