



Name : .....

Roll No. : .....

Invigilator's Signature : .....

**CS/BCA/SEM-2/BM-201/2013**

**2013**

**MATHEMATICS**

Time Allotted : 3 Hours

Full Marks : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

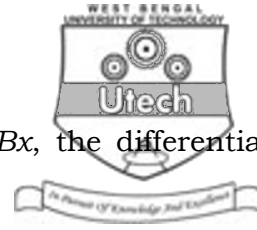
**GROUP – A**

**( Multiple Choice Type Questions )**

1. Choose the correct alternatives for any *ten* of the following :

10 × 1 = 10

- i) A monotonic and bounded sequence is
  - a) convergent
  - b) divergent
  - c) oscillatory
  - d) none of these.
  
- ii) The sequence  $\{r^n\}$  is oscillatory when
  - a)  $r > 1$
  - b)  $r < 1$
  - c)  $-1 < r < 1$
  - d) none of these.



iii) Eliminating  $A$  and  $B$  from  $y = A + Bx$ , the differential equation is obtained as

a)  $\frac{d^2y}{dx^2} + y = 0$

b)  $\frac{d^2y}{dx^2} - y = 0$

c)  $\frac{d^2y}{dx^2} = 0$

d) none of these.

iv) The order and degree of the equation  $\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} = a \frac{dy}{dx}$  is

a) 2, 2

b) 2, 3

c) 3, 2

d) 3, 3.

v) The P.I. of  $\frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + y = e^x$  is

a)  $\frac{e^x}{3}$

b)  $\frac{e^x}{2}$

c)  $\frac{e^x}{6}$

d) none of these.

vi) The series  $\sum_{n=1}^{\infty} n^{\frac{1}{p}}$  is convergent if

a)  $p \geq 1$

b)  $p < 1$

c)  $p > 1$

d)  $p \leq 1$ .



vii) If the series  $\sum_{n=1}^{\infty} u_n$  is convergent, then

- a)  $\lim_{n \rightarrow \infty} u_n = 0$                       b)  $\lim_{n \rightarrow \infty} u_n > 1$
- c)  $\lim_{n \rightarrow \infty} u_n < 1$                       d) none of these.

viii) The series  $1 - 1 + 1 - 1 + \dots$  is

- a) convergent with sum 0
- b) convergent with sum 1
- c) divergent
- d) oscillatory.

ix) The vectors  $(1, 0, 0)$ ,  $(0, 1, 0)$ ,  $(0, 0, 1)$  in  $V_3$  are

- a) linearly dependent                      b) linearly independent
- c) both (a) and (b)                      d) none of these.

x) The basis of a vector space contains

- a) linearly independent vectors
- b) linearly dependent vectors
- c) scalars only
- d) none of these.





4. Test the convergence of the series

$$x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \dots, \quad x > 0$$

5. Define monotone sequence. When is a monotone sequence convergent? Is the following sequence convergent?

$$\left\{ \frac{3n+1}{n+2} \right\}$$

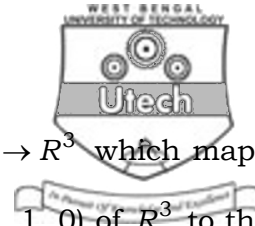
6. Prove that the intersection of two subspaces of a vector space is a subspace.
7. Find the space generated by  $(1, 3, 0)$ ,  $(2, 1, -2)$ . Examine whether  $(4, 7, -2)$  lies in this space.

### GROUP - C

#### ( Long Answer Type Questions )

Answer any *three* of the following.  $3 \times 15 = 45$

8. a) Find the basis and dimension of the subspace  $W$  of  $R^3$  where  $W = \{(x, y, z) \in R^3 : x + y + z = 0\}$ . 5
- b) Test the convergence of the series  $\sum_{n=1}^{\infty} \frac{2^n \cdot n!}{n^n}$ . 5
- c) Solve  $\frac{d^2 y}{dx^2} - 5 \frac{dy}{dx} + 6y = x^2 e^{3x}$ . 5



9. a) Determine the linear mapping  $T : R^3 \rightarrow R^3$  which maps the basis vectors  $(0, 1, 1), (1, 0, 1), (1, 1, 0)$  of  $R^3$  to the vectors  $(1, 2, 1), (1, 1, 2), (2, 1, 1)$  respectively. Find  $\text{Ker} ( T )$  and  $\text{Im} ( T )$ . 8

b) Solve :  $(x^2D^2 - xD - 3)y = x^2 \log x$ . 7

10. a) Define basis and dimension of a vector space. Find a basis and the dimension of  $S \cap T$  where  $S$  and  $T$  are subspaces of  $R^3$  defined by

$$S = \{(x, y, z) \in R^3 : 2x + y + 3z = 0\}$$

$$T = \{(x, y, z) \in R^3 : x + 2y + z = 0\} \quad 2 + 1 + 6$$

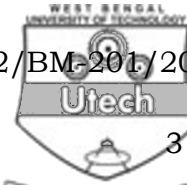
b) Examine whether the vectors  $(1, 2, 2), (2, 1, 2), (2, 2, 1)$  are linearly independent in  $R^3$ . 6

11. a) Test the convergence of the following series :

i)  $\frac{6}{1.3.5} + \frac{8}{3.5.7} + \frac{10}{5.7.9} + \dots$

ii)  $\sum_{n=1}^{\infty} \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{\frac{3}{2}}}$  5 + 5

b) Show that the series  $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$  converges conditionally. 5



3 × 5

12. Solve the following :

a)  $(xy \sin xy + \cos xy) y dx + (xy \sin xy - \cos xy) x dy = 0$

b)  $y = px + \sqrt{a^2 p^2 + b^2}$  ,  $p = \frac{dy}{dx}$

c)  $\frac{d^2y}{dx^2} - y = \sin x$

13. a) Solve  $(x^3 - 3xy^2) dx + (y^3 - 3x^2y) dy = 0$  5

b) Find the representative matrix of the linear transformation  $T : R^3 \rightarrow R^3$  defined by  $T(x, y, z) = (x - 2y, y - 2z, z - 2x)$ . 5

c) Show that  $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots$  is a divergent series. 5

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