BM-301

MATHEMATICS FOR COMPUTING

Time Allotted: 3 Hours | Full Marks: 70

The questions are of equal value.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP A
(Multiple Choice Type Questions)

1. Answer any *ten* questions. $10\times1 = 10$

(i) Maximum number of edges with n vertices in a completely connected graph is

(A) $(n-1)$ (B) $\frac{(n-1)}{2}$

(C) $\frac{n}{2}$ (D) $\frac{n(n-1)}{2}$

(ii) Prim's algorithm is used to find the minimal spanning tree of a

(A) dense graph (B) sparse graph

(C) null graph (D) normal graph

(iii) A Simple graph has

(A) no self loops (B) no parallel edges

(C) both (A) and (B) (D) none of these

(iv) The generating function of $\{1, 1, 1, ...\}$ is

(A) $\frac{1}{(1-x)}$ (B) $\frac{1}{(1+x)}$

(C) $\frac{1}{(1-x)(1+x)}$ (D) none of these

(v) A Grammar is said to be regular if it is of

(A) type 0 (B) type 1
(C) type 2 (D) type 4

(vi) The mathematical model of Mealy machine is a

(A) 5 tuple (B) 4 tuple
(C) 6 tuple (D) none of these

(vii) If $n! = x(n-2)!$ Then $x =$

(A) n (B) $n-1$
(C) $n(n-2)$ (D) $n(n-1)$

(viii) Number of four digit numbers formed by digits 3, 1, 3, 1 is

(A) 5 (B) 10
(C) 20 (D) 6

(ix) The proposition $P \wedge (Q \wedge -Q)$ is a

(A) contradiction (B) tautology
(C) both (A) and (B) (D) none of these

(x) Number of elements contained in an incidence matrix of a digraph is

(A) 1 (B) 2
(C) 3 (D) none of these

(xi) A pseudo graph

(A) Must have loops (B) Does not have loop
(C) Must have parallel edges (D) None of these

(xii) Choose the correct statement:

(A) path is an open walk
(B) every walk is a trail
(C) every trail is a path
(D) a vertex cannot appear twice in a walk

GROUP B
(Short Answer Type Questions)

Answer any *three* questions. $3 \times 5 = 15$

2. In an examination a minimum is to be secured in each of the 5 subjects for a pass. In how many ways can a student fail?

3. Find the sequence from the generating function $\dfrac{(3+7x)}{(1-x)(1+4x)}$

4. Prove that $((P \wedge -Q) \to R) \to (P \to (Q \vee R))$ is a tautology.

5. Write a short note on Moore machine.

6. Show that a tree with n vertices has $(n-1)$ edges.

GROUP C
(Long Answer Type Questions)

Answer any *three* questions. $3 \times 15 = 45$

7. (a) Solve the following recurrence relation by generating function method: 5

$a^n - 2a_{n-1} + a_{n-2} = 2^{n-2}$ for $n \geq 2$ and $a_0 = 1$, $a_1 = 5$

(b) What is a Deterministic Finite Automata (DFA)? Explain with suitable example. 5

(c) Write CNF and DNF of the following statement: $p \vee \{\neg p \to (q \vee (q \to \neg r))\}$ 5

8. (a) Find the grammar of the set of the terminals $\{a, b\}$ that generates the language $L = \{a, ab, ab^2, ab^3, \ldots..\}$. 6

(b) Let Z be a set of all integers and a binary relation ρ is defined on Z by the rule, $m\rho n$ means $m - n$ is divisible by 5 such that ρ is an equivalence relation on Z and identify the equivalent classes. 6

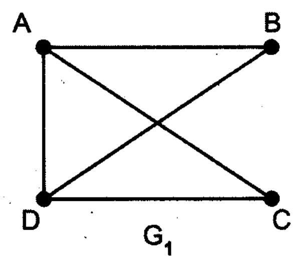
(c) Show that $\underline{|2n} = 2n\{1,3,5\cdots(2n-1)\}\underline{|n}$ 3

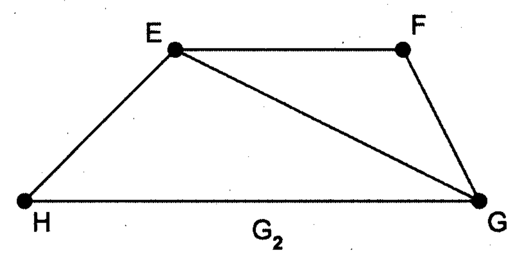
9.(a) Find out the characteristic roots of the recurrence relation $a_n + 4a_{n-1} + 3a_{n-2} = 0$ and hence solve it. 6

(b) Show that a simple graph with n vertices and k components has at most $\frac{(n-k)(n-k+1)}{2}$ edges. 6

(c) Prove the following equivalence: $\neg p \wedge q \Leftrightarrow \neg(p \vee (\neg p \wedge q))$ 3

10.(a) Find whether the following two graphs are isomorphic or not: 5

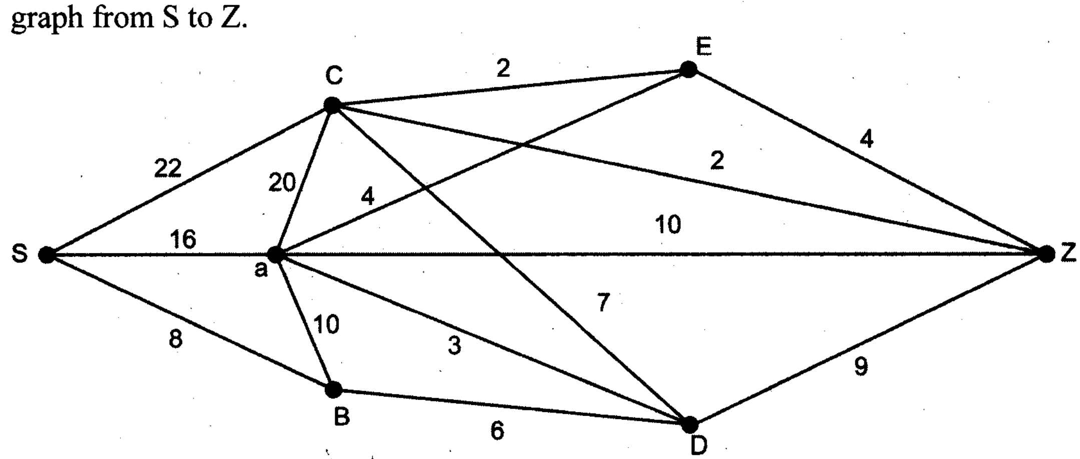




(b) How many integral solutions are there of $x_1 + x_2 + x_3 + x_4 + x_5 = 30$ where $x_1 \geq 2$, $x_2 \geq 3$, $x_3 \geq 4$, $x_4 \geq 2$, $x_5 \geq 0$ 6

(c) Determine the value of n if $4 \times {}^{n}P_3 = {}^{n+1}P_3$. 4

11.(a) Using Kruskal's algorithm find the minimal spanning tree from the following graph from S to Z. 6



(b) Write short note on Hamiltonian Graph. 3

(c) Using the principle of mathematical induction prove that 6

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$