Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/BCA/SEM-3/BM-301/2009-10

2009

MATHEMATICS FOR COMPUTING

Time Allotted : 3 Hours | Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Graph sheet(s) will be provided by the Institution.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The generating function of

{ 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1 } is

a) $\frac{z}{1 + 2z}$ b) $\frac{z}{1 - z^3}$

c) $\frac{1}{1 + z}$ d) $\frac{1}{(1 + z)(1 - z)}$.

ii) How many bit strings of length 10 contain exactly four 1's ?

a) 130 b) 720

c) 386 d) 210.

iii) In how many ways can 5 questions be selected from 8 questions ?

a) 20 b) 40

c) 56 d) 336.

iv) If ${}^{7}P_{x} = 210$, then the value of x is

a) 4 b) 3

c) 8 d) 2.

v) Determine the type of grammar which consists of the productions,

$S \to aAB, \quad AB \to bB, \quad B \to b, \quad A \to aB$

a) Type-1 grammar

b) Regular

c) Type-3 grammar

d) Type-2 grammar.

vi) In a bipartite graph we cannot find a triangle.

a) True b) False.

vii) Express the following sentence in symbolic form :

it is raining but not cloudy.

a) $\sim p \wedge \sim q$ b) $\sim (\sim p \vee q)$

c) $p \wedge -q$ d) none of these.

viii) Let $f : z \to z$ be a mapping defined by $f(x) = 2x - 3$. Then the mapping f is

a) one to one

b) onto

c) neither one-one nor onto

d) both.

ix) The minimum possible height of an n vertex binary tree is

a) $[\log_2 (n-1) + 1]$

b) $[\log_2 (n-1) - 1]$

c) $[\log_2 (n+1) - 1]$

d) $[\log_e (n+1) - 1]$.

x) Let L be a Language given by $L = \{ a^n b^n : n \geq 0 \}$, then L^2 is equal to

a) $\{ a^n b^n a^m b^m : n \geq 0, \ m \geq 0 \}$

b) $\{ a^n b^n : n \geq 0 \}$

c) $\{ a^n b^n a^m b^m : n \geq 0 \}$

d) none of these.

xi) Does there exist a simple graph with 5 vertices of the given degrees ?

1, 2, 3, 4, 5.

a) No

b) Yes

c) Somtime it exists.

xii) In how many ways can the letters of the word UTECH be arranged ?

a) 60 b) 5

c) 120 d) 10.

GROUP – B

(Short Answer Type Questions)

Answer any *three* from the following. 3 × 5 = 15

2. A candidate is required to answer 6 out of 10 questions which are divided into two groups each containing 5 questions and he is permitted to attempt not more than 4 from any group. In how many different ways can he make up his choice ?

3. Find the incidence matrix of the graph :

dia

4. Examine whether the graphs G and G_1 are isomorphic or not.

dia

5. Show that $(2n)! = 2^n \cdot n! \{1, 3, 5 \ldots (2n - 1)\}$.

6. Solve the recurrence relation $a_r - 5a_{r-1} = 3$, $r \geq 1$, with the boundary conditions $a_0 = 1$ using the generating function.

7. In how many ways can three prizes be distributed among 4 boys when

   i) no one gets more than one prize ?

   ii) a boy can get any number of prizes ?

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Solve the following equivalence recurrence relation using generating functions :

   $a_n - 2a_{n-1} + a_{n-2} = 2^{n-2}$ for $n \geq 2$ & $a_0 = 1$, $a_1 = 5$.

   b) Let Z be the set of all integers and a binary relation ρ is defined on Z by the rule, $m \rho n$ means $m - n$ is divisible by 5 such that ρ is an equivalence relation on Z and identify all equivalent classes.

9. a) Draw the graph represented by the given adjacency matrix.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

   b) Show that a tree of n vertices has $n - 1$ edges.

c) Write short notes on 'Mealy Machine' and 'Moore Machine'.

10. a) Show that a simple graph with n vertices and k components has at most

$(n - k) (n - k + 1) / 2$ edges.

b) Using Kruskal's algorithm find a spanning tree with minimum weight from the graph given below. Also calculate the total weight of spanning tree.

dia

11. a) Examine if the following two graphs are isomorphic :

dia

b) Construct the state diagram for finite state machine with state tables as under :

State	f		G	
	Input		Output	
	0	1	0	1
S_0	S_1	S_0	1	0
S_1	S_3	S_0	1	0
S_2	S_1	S_0	0	1
S_3	S_2	S_1	0	0

c) Write short notes on any *two* of the following :

i) Spanning Graph

ii) Hamiltonian Graph

iii) Digraph.

12. a) Solve the equation $2x^3 - x^2 - 22x - 24 = 0$, two of the roots being in the ratio 3 : 4.

b) Let $R = \left\{ \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} ; \alpha \in R \right\}$, show that R is an Abelian group with respect to usual matrix multiplication.

c) In a survey of entertainment habits of 1000 persons, it was found that 400 persons do not see cinema or see TV. 500 persons go to cinema and 200 persons see TV. Find out the number of persons who see cinema as well as TV. Find also the number of persons who see only TV but not cinema and those who go to cinema but not see TV. 5 + 5 + 5

13. a) Evaluate :

i) $\int \frac{1 + \sin x}{\sin x \, (1 + \cos x)} \, dx$

ii) $\int_0^{\pi} x \log \sin x \, dx.$

b) Find the point of intersection of the straight lines

$\frac{K}{r} = \cos \theta - \cos (\theta - \alpha)$ and

$\frac{K}{r} = \cos \theta - \cos (\theta - \beta)$. (5 + 5) + 5

14. a) Discuss the nature of the conic represented by

$$x^2 + 2xy + y^2 - 4x - 4y + 3 = 0$$

by reducing to its canonical form.

b) If PSQ be a focal chord of a conic with focus S and semi-latus rectum l then prove that

$\frac{1}{SP} + \frac{1}{SQ} = \frac{2}{l}$. 9 + 6

15. a) Find the point on the conic $\frac{9}{r} = 3 - 5 \cos \theta$ which has the smallest radius vector.

b) If $u = \frac{x^2 + y^2}{\sqrt{xy}}$, $(x, y) \neq (0, 0)$ and

$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = ku$, find the value of k.

c) Use mean-value theorem to prove the following inequality :

$$0 < \frac{1}{x} \log \frac{e^x - 1}{x} < 1.$$ 4 + 5 + 6