Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/BCA/SEM-4/BM-401/2010

2010

STATISTICS, NUMERICAL & METHODS & ALGORITHMS

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) If $f(x)$ is a polynomial of degree n, then ..... is a constant.

      a) $(n+1)$th order difference

      b) nth order difference

      c) $(n-1)$th order difference

      d) $(n-2)$th order difference.

   ii) One of the roots of the equation $x^2+2x-2=0$ lies in between

      a) 1 & 2 b) 0 & 0·5

      c) 0·5 & 1·0 d) none of these.

iii) First order forward difference of a constant function is

a) 0 b) 4

c) 3 d) 1.

iv) Inverse of a matrix A is given by

a) $A^{-1} = \frac{adj\ A}{det\ A}$ b) $A^{-1} = \frac{det\ A}{adj\ A}$

c) $A^{-1} = (det\ A)^T$ d) $A^{-1} = (adj\ A)^T$.

v) $\delta E^{\frac{1}{2}}$ is equal to

a) ∇ b) Δ

c) E d) none of these.

vi) The inherent error in the Runge-Kutta method is of order

a) h^2 b) h^4

c) h^5 d) h^6.

vii) The value of $\left(\frac{\Delta^2}{E}\right)x^4$ is

a) $6x$ b) $6x^2$

c) $6x^3$ d) $6x^0$.

viii) If E is the shift operator and Δ is the forward difference operator, then relationship between them is

a) $E = \Delta + 1$ b) $E = \Delta^{-1}$

c) $E + 1 = \Delta$ d) none of these.

ix) Let $f(x) = 0$ be the equation of a curve. Then the condition that one of the roots of $f(x)$ lies between $x = a$ and $x = b$ is

a) $f(a) > 0$ b) $f(a)\, f(b) < 0$

c) $f(a)\, f(b) > 0$ d) none of these.

x) Simpson's $\frac{1}{3}$rd rule gives us exact result for a polynomial of degree

a) less than 3

b) less than equal to 3

c) greater than 3

d) greater than equal to 3.

xi) If $u_0 = 1$, $u_1 = 1$ and $u_2 = 21$, then $\Delta^2 u_0$ is

a) 10 b) 11

c) 0 d) 20.

xii) By evaluating $\int_0^{\infty} \frac{dx}{1+x^2}$ by numerical integration method, we can obtain the approximate value of

a) $log_e 2$ b) $\frac{\pi}{2}$

c) e d) $log_{10} 2$.

xiii) For a system of equation $Ax = b$, a solution exists if and only if A is

a) symmetric b) singular

c) orthogonal d) diagonal.

xiv) Equation $AX = B$ has unique solution if

a) Rank (A) ≠ Rank ($A\ B$)

b) Rank (A) < Rank ($A\ B$)

c) Rank (A) = Rank ($A\ B$) = No. of unknowns

d) Rank (A) = Rank ($A\ B$) ≠ No. of unknowns.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. 3 × 5 = 15

2. Prove that $D = \frac{1}{h} log \frac{1}{(1-\nabla)}$, where D is differential operator and ∇ is backward difference operator.

3. Find the value of $\frac{dy}{dx}$ for x = 1·0 from the following table :

x :	1·0	1·2	1·4	1·6	1·8	2·0
y :	2·7183	3·3201	4·0552	4·9530	6·0496	7·3891

4. Find a root of the equation $x^3 - 3x - 5 = 0$ by the method of false position correct to 2 decimal places.

5. Using Taylor's method obtain an approximate value of y at x = 0·2 for the differential equation $\frac{dy}{dx} = 2y + 3e^x$, y (0) = 0.

6. Solve the system of equations by Gauss elimination method :

$2x + 3y + z = 9$

$x + 2y + 3z = 6$

$3x + y + 2z = 8$

correct upto three significant figures.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Evaluate y (1·1) using Runge-Kutta method of order 4 for the problem

$$\frac{dy}{dx} = x^2 + y^2, \quad y(1) = 0$$

b) Find the inverse of the matrix $\begin{bmatrix} 1 & 2 & 6 \\ 2 & 5 & 15 \\ 6 & 15 & 46 \end{bmatrix}$ by Gauss elimination method.

8. a) Compute f (0·29) from the following table by using Newton's backward interpolation formula :

x :	0·20	0·22	0·24	0·26	0·28	0·30
y :	1·6596	1·6698	1·6804	1·6912	1·7024	1·7139

b) The following are the mean temperature (Fahrenheit) on the three days, 30 days apart round the periods of summer and winter. Estimate the approximate dates and the values of the maximum dates and the values of the maximum and minimum temperature.

Day	Summer		Winter	
	Date	Temperature	Date	Temperature
0	15th June	58·8	16th December	40·7
30	15th July	63·4	15th January	38·1
60	14th August	62·5	14th February	39·3

9. a) Using Newton's divided difference formula, construct the interpolation polynomial and hence compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 5$ by using the following data :

x :	0	2	3	4	7	9
y :	4	26	58	112	466	922

b) Evaluate $\int_0^1 x^3 \, dx$ by Trapezoidal rule with $n = 5$.

10. a) Evaluate one root of the following equation, by Newton – Raphson method :

$e^x - 3x = 0$

correct up to 3 decimal places.

b) Use Euler's method to find the numerical solution of the following differential equation :

$f'(x) = 1 + x - x^2$, y (0) = 1, h = 0·02;

find y (0·1).

11. a) Find the missing term in the following table :

x :	0	1	2	3	4	5
y :	0	—	8	15	—	35

b) What is the lowest degree polynomial which takes the following values :

x :	0	1	2	3	4	5
$f(x)$:	1	4	9	16	25	36

Hence calculate $f(x)$ and also find $f(6)$.