

Name :

Roll No. :

Invigilator's Signature :

CS/B.Pharm/SEM-1/M-103/2009-10

2009

REMEDIAL MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any ten of the following : $10 \times 1 = 10$

i) If A is a square matrix and A' its transpose, then

$A + A'$ is

- a) a skew-symmetric matrix
- b) a diagonal matrix
- c) a unit matrix
- d) a symmetric matrix.

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- ii) $\int_{-2}^2 x^3 \cos x \, dx$ is equal to

 - a) 4
 - b) 2
 - c) 0
 - d) none of these.

iii) The order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 + y = 0$ is

 - a) 3
 - b) 2
 - c) 4
 - d) none of these.

iv) $\lim_{x \rightarrow \pi} \frac{\sin x}{\pi - x}$ is equal to

 - a) 0
 - b) $\frac{1}{2}$
 - c) 1
 - d) -1.

v) A matrix is said to be singular if

 - a) $\det A \neq 0$
 - b) $\det A = 0$
 - c) $\text{adj } A \neq 0$
 - d) $\text{adj } A = 0$.

vi) If $f(x) = |x|$, then $f'(x) =$

 - a) 2
 - b) 1
 - c) 0
 - d) does not exist.



vii) $\lim_{x \rightarrow \infty} (1+x)^{\frac{1}{x}}$ is equal to

- a) e b) $\frac{1}{e}$
c) $\frac{1}{e^2}$ d) 1.

viii) If C be a scalar, then $(CA)^T =$

- a) $C^T A^T$ b) $A^T C^T$
c) CA^T d) none of these.

ix) Let $\Delta = \begin{vmatrix} 0 & a-b & a-c \\ b-c & 0 & b-c \\ c-a & c-b & 0 \end{vmatrix}$ then the value of Δ is

- a) 0
b) 1
c) abc
d) $(a-b)(b-c)(c-a)$.

x) The degree of the differential equation $\frac{d^2y}{dx^2} = \left(R^{\frac{1}{3}} \frac{dy}{dx}\right)^{\frac{2}{3}}$

- is
a) $2/3$ b) 3
c) 2 d) 1.



xii) Every diagonal element of a skew-symmetric matrix is

xii) $\int e^x (\cos x - \sin x) dx$ is equal to

- a) $e^x + c$ b) $e^x \cos x + c$
c) $e^x \sin x + c$ d) $\cos x \sin x + c.$

xiii) The value of $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$ is

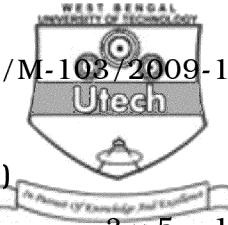
xiv) Let A be a matrix of order $m \times p$ and B of order $p \times q$.

Then AB is a matrix of order

- a) $p \times q$ b) $m \times p$
 c) $p \times p$ d) $m \times q$.

xy) If $x = r \cos \theta$ and $y = r \sin \theta$, then $x \, dx + y \, dy$ is

- a) $r \, dr$ b) $r \, d\theta$
 c) r d) θ



GROUP - B
(Short Answer Type Questions)

Answer any *three* of the following.

$$3 \times 5 = 15$$

2. Find the matrices A and B if $2A + 3B = \begin{pmatrix} 8 & 3 \\ 7 & 6 \end{pmatrix}$,

$$A + B^T = \begin{pmatrix} 3 & 1 \\ 3 & 3 \end{pmatrix}.$$

3. Show that $\lim_{x \rightarrow 0} \frac{x^2}{x} = 0$.

4. If $y = \sqrt{3x+2}$, prove that $y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 = 0$.

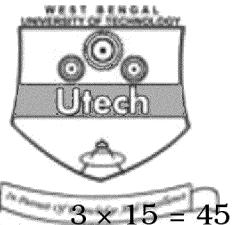
5. Prove that $\begin{vmatrix} b+c & c+a & a+b \\ q+r & r+p & p+q \\ y+z & z+x & x+y \end{vmatrix} = 2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix}$.

6. If $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$, show that $A^n = \begin{pmatrix} 1+2n & -4n \\ n & 1-2n \end{pmatrix}$.

7. Evaluate $\int_0^4 \frac{dx}{x + \sqrt{16-x^2}}$.

8. Find the differential equation of the system of circles

touching y -axis at the origin.



GROUP – C
(Long Answer Type Questions)
 Answer any *three* of the following.

9. a) Solve $4^x - 3 \cdot 2^{x+2} + 2^5 = 0$

b) Prove that
$$\begin{vmatrix} a-b-c & 2b & 2c \\ 2a & b-c-a & 2c \\ 2a & 2b & c-a-b \end{vmatrix} = (a+b+c)^3.$$

c) Evaluate $\int \tan^{-1} \sqrt{x} dx.$

10. a) Solve by Cramer's rule :

$$2x - y = 3, 3y - 2z = 5, 2z - x = -4.$$

b) Prove that

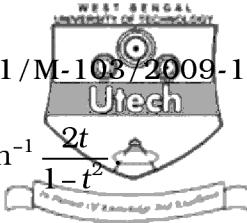
$$\lim_{n \rightarrow \infty} \left[\frac{1}{n} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + \dots + \frac{1}{8n} \right] = \frac{7}{16}.$$

c) Show that every square matrix can be expressed as the sum of a symmetric and a skew symmetric matrix.

11. a) Find the matrix A if $\text{adj } A = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 5 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ and $\det A = 2$.

b) State Rolle's theorem. Verify Rolle's theorem for the function $f(x) = x(x-1)(x-2)$ in $0 < x < 2$.

c) Evaluate : $\int \sqrt{\tan} dx.$



12. a) Find $\frac{dy}{dx}$ if $y = \sin^{-1} \frac{2t}{1+t^2}$ and $x = \tan^{-1} \frac{2t}{1-t^2}$.
- b) Solve $x \frac{dy}{dx} + y = y^2 \log x$.
- c) Show that $\int_0^\pi \frac{x \sin x}{1+\cos^2 x} dx = \frac{\pi^2}{4}$.
13. a) If $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$, find A^{-1} .
- b) Find the values of a , b and c so that $\lim_{x \rightarrow 0} \left[\frac{ae^x - b\cos x + ce^{-x}}{x \sin x} \right] = 2$.
- c) If $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, examine whether $AB = BA$ or not.
14. a) Find the maximum or minimum value of $x^{\frac{1}{x}}$.
- b) If $y = (x^2 - 1)^n$, then show that $(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$.
- c) Solve $\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = e^{-x}$.