Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS / B.PHARM / SEM-1 / M-103/ 2010-11

2010-11

REMEDIAL MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i) The value of $\int \frac{dx}{x \log x} =$

a) $\log(\log x)$ b) $\log x$

c) $1/\log x$ d) none of these.

ii) $f(x, y) = (x^3 - y^3)/(x + y)$ is a homogeneous function of degree

a) 0 b) 1

c) 2 d) 3.

iii) The determinant $\begin{vmatrix} 3x^2 & 3x & 1 \\ x^2 + 2x & 2x + 1 & 1 \\ 2x + 1 & x + 2 & 1 \end{vmatrix}$ has a factor

a) x b) $x + 3$

c) $x - 1$ d) $2x + 1$.

iv) If $A = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}$, then $A^{23} =$

a) $-A$ b) A

c) I_2 d) none of these.

v) The matrix $\begin{bmatrix} 1 & 2 & 3 \\ -2 & 0 & 4 \\ -3 & -4 & 5 \end{bmatrix}$ is

a) symmetric b) skew-symmetric

c) singular d) none of these.

vi) If $A = \begin{bmatrix} 7 & 6 & -3 \\ 8 & 14 & -4 \\ 4 & 7 & \lambda \end{bmatrix}$ is a singular matrix, then λ = ?

a) 4 b) 5

c) -4 d) none of these.

vii) The value of the limit $\underset{x \to \frac{\pi}{2}}{Lt} \frac{\cos x}{\frac{\pi}{2} - x}$ is

a) 1 b) 0

c) -1 d) none of these.

viii) The function $f(x) = \log\left(x + \sqrt{1 + x^2}\right)$ is a/an

a) odd function b) even function

c) periodic function d) none of these.

ix) The value of $\lim_{x \to 0} (1 + x)^{\frac{1}{2x}}$ is

a) 1 b) $\sqrt{e}$

c) does not exist d) 0.

x) If $y = e^{mx}$, then $y_n =$

a) e^{mx} b) $e^{mx} m^n$

c) $\angle n\, e^{mx}$ d) none of these.

xi) If $\varphi'(x) = \psi'(x)$ in (a, b), then

a) $\varphi(x) = \psi(x)$ in (a, b)

b) $\varphi(x)\,\psi(x) = 0$ in (a, b)

c) $\varphi(x) = \psi(x) +$ constant in (a, b)

d) none of these.

xii) The degree of the ordinary differential equation $\left(\frac{d^2y}{dx^2}\right)^2 = \sqrt{\frac{dy}{dx} - y}$ is

a) 2 b) $\frac{1}{2}$

c) 4 d) None of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Without expanding prove that $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix} = 0$

3. A function is defined as follows : *e* function at $x = 0$ =

$F(x) = -x$ when $x < 0$ $= x$ when $0 < x < 1$

$= 2 - x$ when $z >= 1$

Discuss the continuity of the function at $x = 0$ and $x = 1$.

4. If $v = z\tan^{-1} y/x$, show that $v_{xx} + v_{yy} + v_{zz} = 0$.

5. Form the differential equation of the family of curves $y^2 = 4a(x + a)$, a being the parameter.

6. Evaluate : $\int \frac{x^3 dx}{(x+1)(x+2)}$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Solve the following equation by matrix method :

$x + y - z = 6$

$2x - 3y + z = 1$

$2x - 4y + 2z = 1$

b) Differentiate

$\tan^{-1}\sqrt{(1 + x^2 - 1)}/x$ with respect to $\tan^{-1} x$.

c) Solve : $dx - (xy + x^2y^3)\,dy = 0$ 6 + 5 + 4

8. a) Discuss the derivability of the function at $x = 2$.

$$f(x) = \begin{cases} x, & x < 1 \\ 2 - x, & 1 \le x \le 2 \\ -2 + 3x - x^2, & x > 2 \end{cases}$$

b) If $y = x^{n-1} \log x$, show that $y_n = \dfrac{(n-1)!}{x}$.

c) Verify that $\dfrac{\partial^2 u}{\partial x \, \partial y} = \dfrac{\partial^2 u}{\partial y \, \partial x}$ where $u = \dfrac{x}{y} + \dfrac{y}{x}$. 5 + 5 + 5

9. a) Solve : $\dfrac{dy}{dx} = e^{x+y} - 1$.

b) Obtain the complete primitive and the singular solution of the equation : $y = px + \sqrt{1 + p^2}$.

c) Solve : $y - x\dfrac{dy}{dx} = 2\left(1 + x^2 \dfrac{dy}{dx}\right)$. 4 + 5 + 6

10. a) Solve : $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 2y = 0$ when $x = 0$, $y = 3$ and $\dfrac{dy}{dx} = 0$.

b) Find the equation of the curve having slope at any point as $y + 2x$ and passing through (0, 0).

c) Solve : $(D^2 + 2D)\, y = x^2$.

d) Solve : $\dfrac{dy}{dx} + \dfrac{1}{x} y = y^2$. 4 + 5 + 3 + 3

11. a) State Lagrange's M.V.T. Give its geometrical interpretation. Verify Lagrange's M.V.T. for $f(x) = x^2 - 4x + 9$ in $2 \le x \le 3$.

b) Find the maxima and minima of the function $f(x) = 1 + 2\sin x + 3\cos^2 x$ on $0 < x < \frac{\pi}{2}$.

c) State Euler's theorem on homogeneous function in two variables. If $u = \tan^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{\sin 2u}{4}$. 6 + 4 + 5

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