Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/B.PHARM/SEM-1/M-103/2011-12

2011

REMEDIAL MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) If $y = x^n$, n being a positive integer, then y_{n-1} =

a) $(n-1).x$ b) $\frac{\lfloor n}{\lfloor 1}.x$

c) $\lfloor n-1$ d) none of these.

ii) $\lim_{x \varnothing a} \frac{x-a}{|x-a|}$ =

a) 1 b) -1

c) 0 d) none of these.

iii) If $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$, then

a) A^{-1} exists b) $A = -I_3$

c) A is a null matrix d) none of these.

iv) $\begin{vmatrix} a+b & a+2b & a+3b \\ a+2b & a+3b & a+4b \\ a+4b & a+5b & a+6b \end{vmatrix} =$

a) $a^3 + b^3 + c^3$

b) $a^2 + b^2 + c^2 - ab - bc - ca$

c) 0

d) none of these.

v) $f(x, y) = \dfrac{\sqrt[3]{x} + \sqrt[3]{y}}{\sqrt{x} - \sqrt{y}}$ is a homogeneous function in x and y of degree

a) $\dfrac{1}{6}$ b) $-\dfrac{1}{6}$

c) $\dfrac{1}{4}$ d) none of these.

vi) The relation $y = Ae^{x} + Be^{-x}$, A and B being parameters gives rise to a differential equation of order

a) 2 b) 1

c) 3 d) none of these.

vii) If $f(x) = \dfrac{|x|}{x}$, then $f(0) =$

a) 0 b) 1

c) -1 d) none of these.

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viii) If $f(x)$ is continuous in $[x, x+h]$, derivable in $(x, x+h)$, then $f(x+h) = f(x) + hf'(x+\theta h)$ where

a) θ is any rational number

b) θ is a positive proper fraction

c) θ is an integer

d) none of these.

ix) $\int_{-a}^{a} x\sqrt{x^2 - a^2}\, dx =$

a) 3 b) -3

c) 0 d) none of these.

x) If $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 3-x & x^2-4 \\ 0 & 0 & 4 \end{pmatrix}$ is a singular matrix, then x is

a) 4 b) 1

c) 2 d) none of these.

xi) $Det(I_3)$ is

a) 6 b) 1

c) 2 d) 36.

xii) $\dfrac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}}$ is a homogeneous function of degree

a) 0 b) 1

c) $\frac{1}{2}$ d) none of these.

xiii) Rank of $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{pmatrix}$ is

a) 0 b) 1

c) 2 d) 3.

xiv) Order of the differential equation

$\left(\frac{dy}{dx}\right)^2 - \frac{1}{x}\left(\frac{dy}{dx}\right)^3 = -\frac{1}{x^2}$ is

a) 3 b) 1

c) 2 d) none of these.

xv) Degree of the differential equation

$x \cos x \left(\frac{dy}{dx}\right)^2 + y^3 (x \sin x + \cos x) = x^9$ is

a) 3 b) 1

c) 2 d) 0.

xvi) The differential equation $y\,dx - x\,dy = x^2\, y\,dx$ is exact.

a) True b) False.

GROUP – B

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(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Find the value of K if $\begin{pmatrix} 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \\ K \end{pmatrix} = 0$.

3. Prove that

$$\begin{vmatrix} a & b+c & a^2 \\ b & c+a & b^2 \\ c & a+b & c^2 \end{vmatrix} = -(a+b+c)(a-b)(b-c)(c-a).$$

4. If $\lim\limits_{x \to 0} \dfrac{\sin 2x + a \sin x}{x^3}$ be finite, find the value of a and the limit.

5. Evaluate $\displaystyle\int_0^{\pi/2} \frac{2 \sin x + 3 \cos x}{\sin x + \cos x}\, dx$.

6. Varify Euler's theorem for the homogeneous function $z = e^{\frac{x}{y}}$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Evaluate the following :

i) $\displaystyle\int \frac{dx}{1 + \tan x}$

ii) $\displaystyle\int \frac{x^2}{(x+1)(x+2)}\, dx$.

b) Solve : $4^x - 3^{x - \frac{1}{2}} = 3^{x + \frac{1}{2}} - 2^{2x-1}$.

c) If $V = (ax + by)^2 - (x^2 + y^2)$ and $a^2 + b^2 = 2$, show that $V_{xx} + V_{yy} = 0$. 5 + 5 + 5

8. a) In the Mean Value Theorem

$$f(x + h) = f(x) + hf'(x + \theta h),$$

find θ where $f(x) = e^x$.

b) If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, find A^{-1}.

c) Find the differential equation of the system of circles $(x - c)^2 + y^2 = a^2$, c being the parameter.

5 + 5 + 5

9. a) State Rolle's theorem. Verify Rolle's theorem for the function $f(x) = 2x^3 + x^2 - 4x - 2$, $-\sqrt{2} \le x \le \sqrt{2}$.

b) Use Mean value theorem to prove the following inequality :

$$0 < \frac{1}{x} \log \frac{e^x - 1}{x} < 1.$$ 8 + 7

10. a) Solve the system of equations by Cramer's rule :

$$x + 2y - 3z = 1$$

$$2x - y + z = 4$$

$$x + 3y = 5.$$

b) If $u = \cos^{-1} \frac{x + y}{\sqrt{x} + \sqrt{y}}$, then prove that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0.$$ 8 + 7

11. a) Find the maximum and minimum values of

$y = (1 - x)^2 e^x$.

b) Prove that

$$\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = 4 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

c) Show that

$$\lim_{n \to \infty} \left[\frac{1}{n+m} + \frac{1}{n+2m} + \dots + \frac{1}{n+nm} \right] = \frac{1}{m} \log (1 + m).$$

5 + 5 + 5

12. a) Solve : $2xy \, dx - (x^2 - y^2) \, dy = 0$.

b) Solve : $(D^2 - 4D + 4) \, y = x^3 e^{2x}$.

c) Find the equation of the curve of which slope at any point (x, y) is xy and which passes through the point (0, 1). 5 + 5 + 5