

Name :

Roll No. :

Invigilator's Signature :

CS / B.PHARM / SEM-1 / M-103 / 2012-13

2012

REMEDIAL MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP - A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10

i) In Lagrange's mean value theorem $f(x)$ should be continuous in

- a) Closed interval b) Open interval
- c) Semi-open interval d) None of these.

ii) $f(x) = \left(\sqrt{xy} + y^{\frac{1}{4}} \right)^3$ is a homogeneous function of degree

- a) 1 b) 2
- c) $\frac{3}{4}$ d) $\frac{1}{4}$.



iii) If $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 3-x & x^2-4 \\ 2 & 0 & 4 \end{pmatrix}$ is a singular matrix, then x is

- a) 4
- b) 1
- c) 2
- d) none of these.

iv) $\det(I_{36})$ is

- a) 6
- b) 1
- c) 2
- d) 36.

v) If $x = r \cos \theta$ and $y = r \sin \theta$, then $xdx + ydy$ is equal to

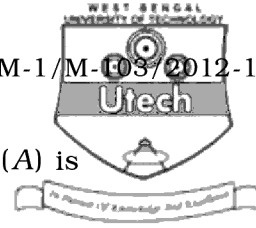
- a) rdr
- b) $rd\theta$
- c) θ
- d) r .

vi) Rank of $\begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{pmatrix}$ is

- a) 0
- b) 1
- c) 2
- d) 3.

vii) Value of $\begin{vmatrix} -1 & 2 & 3 & -4 \\ 3 & 4 & 9 & 8 \\ 2 & 4 & 6 & 0 \\ 1 & -2 & -3 & 4 \end{vmatrix}$ is

- a) 0
- b) 68
- c) 1
- d) none of these.



viii) If A be an orthogonal matrix, the $\det (A)$ is

- a) 0
- b) 1
- c) 1 or - 1
- d) none of these.

ix) $\lim_{n \rightarrow 0} (1 + x)^{\frac{1}{2^n}}$ equals to

- a) 1
- b) \sqrt{e}
- c) 0
- d) does not exist.

x) $\frac{\partial}{\partial y} (x^y) =$

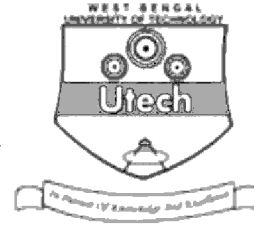
- a) 0
- b) yx^{y-1}
- c) $x^y \log x$
- d) none of these.

xi) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{\frac{\pi}{2} - x}$ is

- a) 1
- b) 0
- c) - 1
- d) None of these.

xii) $\int e^x (\cos x - \sin x) dx$ is equal to

- a) $e^x + c$
- b) $e^x \cos x + c$
- c) $e^x \sin x + c$
- d) $\cos x \sin x + c.$



xiii) The degree of the differential equation

$$y = x \frac{dy}{dx} + c \frac{dx}{dy} \text{ is}$$

- a) 1
- b) 2
- c) 3
- d) 4.

xiv) Order of the ordinary differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 = \sqrt{\frac{dy}{dx} - y} \text{ is}$$

- a) 2
- b) 4
- c) $\frac{1}{2}$
- d) none of these.

GROUP - B

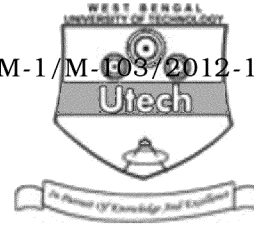
(Short Answer Type Questions)

Write short notes on any *three* of the following. $3 \times 5 = 15$

2. Without expansion prove that

$$\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix} = 0$$

3. If $\sin y = x \sin (a + y)$, prove that $\frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$.



4. If $f(x, y) = x \cos y + y \cos x$, show that

$$f_{xy} = f_{yx}.$$

5. Solve $\frac{dy}{dx} - \frac{1}{x}y = -\frac{1}{x^2}$.

6. Find the integrating factor (IF) of the differential equation $2 \sin y^2 dx + xy \cos y^2 dy = 0$.

GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Prove that $\begin{vmatrix} a & -b & -a & b \\ b & a & -b & -a \\ c & -d & c & -d \\ d & c & d & c \end{vmatrix} = 4(a^2 + b^2)(c^2 + d^2).$ 7

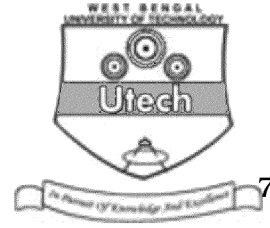
b) If $f(x, y) = \frac{x^2y^2}{x^2 + y^2}$, when $(x, y) \neq (0, 0)$

$= 0$, when $(x, y) = (0, 0)$

find $f_x(0, 0)$ and $f_y(0, 0)$. 8

8. a) If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, prove that $A^3 - 2A + I_3 = O_3$. Hence

find A^{-1} . 8



b) Find the rank of $\begin{pmatrix} 2 & 3 & -1 & 1 \\ 3 & 0 & 4 & 2 \\ 6 & 9 & -3 & 3 \end{pmatrix}$.

7

9. a) If $x = r \cos \theta$, $y = r \sin \theta$, Find $\frac{\partial (x, y)}{\partial (r, \theta)}$. 5

b) State Euler's theorem for homogeneous function. 2

c) If $u = \sin^{-1} \sqrt{\frac{\frac{1}{x^3} + \frac{1}{y^3}}{\frac{1}{x^2} - \frac{1}{y^2}}}$ then verify whether the following

identity is true :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12} \left(\frac{13}{12} + \frac{\tan^2 u}{12} \right). \quad 8$$

10. a) Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{1 + \cot x}$.

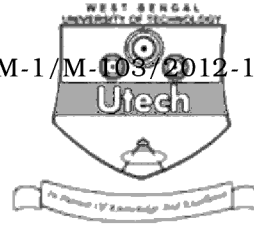
b) Solve : $(D - 2)^2 y = 8 \sin 2x$, where $D \equiv \frac{d}{dx}$.

c) Solve : $(3x^2y^4 + 2xy) dx + (2x^3y^3 - x^2) dy = 0$.

11. a) Form the differential equation of the family of curves

$$y = a \sin (\omega t + b), \text{ where } a \text{ and } b \text{ are arbitrary}$$

constants.



- b) In the Mean Value Theorem

$$f(a+h) = f(a) + hf'(a+\theta h), \text{ if } a = 1, h = 3 \text{ and } f(x) = \sqrt{x}, \text{ find } \theta.$$

- c) Solve $5^{x+1} + 5^{2-x} = 5^3 + 1$.

12. a) If $y = (x^2 - 1)^n$, then show that

$$(x^2 - 1) y_{n+2} + 2x y_{n+1} - n(n+1) y_n = 0.$$

- b) If $f(x) = 2 - x$, when $1 \leq x \leq 2$

$$= x - \frac{1}{2}x^2, \text{ when } x > 2, \text{ prove that } f(x) \text{ is}$$

continuous at $x = 2$.

- c) Show that the minimum value of $4e^{2x} + 9e^{-2x}$ is 12.

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