Name : ....

Roll No. : ....

Invigilator's Signature : ....

2012

REMEDIAL MATHEMATICS

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) In Lagrange's mean value theorem $f(x)$ should be continuous in

   a) Closed interval b) Open interval

   c) Semi-open interval d) None of these.

   ii) $f(x) = \left(\sqrt{xy} + y^{\frac{1}{4}}\right)^3$ is a homogeneous function of degree

   a) 1 b) 2

   c) $\frac{3}{4}$ d) $\frac{1}{4}$.

iii) If $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 3-x & x^2-4 \\ 2 & 0 & 4 \end{pmatrix}$ is a singular matrix, then x is

a) 4 b) 1

c) 2 d) none of these.

iv) det (I_{36}) is

a) 6 b) 1

c) 2 d) 36.

v) If $x = r\cos\theta$ and $y = r\sin\theta$, then $x\,dx + y\,dy$ is equal to

a) $r\,dr$ b) $r\,d\theta$

c) θ d) r.

vi) Rank of $\begin{pmatrix} 0 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{pmatrix}$ is

a) 0 b) 1

c) 2 d) 3.

vii) Value of $\begin{vmatrix} -1 & 2 & 3 & -4 \\ 3 & 4 & 9 & 8 \\ 2 & 4 & 6 & 0 \\ 1 & -2 & -3 & 4 \end{vmatrix}$ is

a) 0 b) 68

c) 1 d) none of these.

viii) If A be an orthogonal matrix, the det (A) is

a) 0 b) 1

c) 1 or – 1 d) none of these.

ix) $\underset{n \to 0}{Lt} (1 + x)^{\frac{1}{2x}}$ equals to

a) 1 b) $\sqrt{e}$

c) 0 d) does not exist.

x) $\frac{\partial}{\partial y}(x^y) =$

a) 0 b) yx^{y-1}

c) $x^y \log x$ d) none of these.

xi) $\underset{x \to \frac{\pi}{2}}{Lt} \frac{\cos x}{\frac{\pi}{2} - x}$ is

a) 1 b) 0

c) – 1 d) None of these.

xii) $\int e^x (\cos x - \sin x)\, dx$ is equal to

a) $e^x + c$ b) $e^x \cos x + c$

c) $e^x \sin x + c$ d) $\cos x \sin x + c$.

xiii) The degree of the differential equation

$$y = x\frac{dy}{dx} + c\frac{dx}{dy} \text{ is}$$

a) 1 b) 2

c) 3 d) 4.

xiv) Order of the ordinary differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 = \sqrt{\frac{dy}{dx} - y} \text{ is}$$

a) 2 b) 4

c) $\frac{1}{2}$ d) none of these.

GROUP – B

(Short Answer Type Questions)

Write short notes on any *three* of the following. 3 × 5 = 15

2. Without expansion prove that

$$\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix} = 0$$

3. If sin $y = x$ sin ($a + y$), prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$.

4. If $f(x, y) = x \cos y + y \cos x$, show that

$f_{xy} = f_{yx}$.

5. Solve $\frac{dy}{dx} - \frac{1}{x} y = -\frac{1}{x^2}$.

6. Find the integrating factor (IF) of the differential equation $2 \sin y^2 \, dx + xy \cos y^2 \, dy = 0$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Prove that $\begin{vmatrix} a & -b & -a & b \\ b & a & -b & -a \\ c & -d & c & -d \\ d & c & d & c \end{vmatrix} = 4(a^2 + b^2)(c^2 + d^2)$. 7

b) If $f(x, y) = \frac{x^2 y^2}{x^2 + y^2}$, when $(x, y) \neq (0, 0)$

$= 0$, when $(x, y) = (0, 0)$

find $f_x(0, 0)$ and $f_y(0, 0)$. 8

8. a) If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$, prove that $A^3 - 2A + I_3 = O_3$. Hence find A^{-1}. 8

b) Find the rank of $\begin{pmatrix} 2 & 3 & -1 & 1 \\ 3 & 0 & 4 & 2 \\ 6 & 9 & -3 & 3 \end{pmatrix}$. 7

9. a) If $x = r\cos\theta$, $y = r\sin\theta$, Find $\dfrac{\partial(x, y)}{\partial(r, \theta)}$. 5

b) State Euler's theorem for homogeneous function. 2

c) If $u = \sin^{-1}\sqrt{\dfrac{x^{\frac{1}{3}} + y^{\frac{1}{3}}}{x^{\frac{1}{2}} - y^{\frac{1}{2}}}}$ then verify whether the following identity is true :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right).$$ 8

10. a) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{1 + \cot x}$.

b) Solve : $(D-2)^2\, y = 8\sin 2x$, where $D \equiv \dfrac{d}{dx}$.

c) Solve : $(3x^2y^4 + 2xy)\, dx + (2x^3y^3 - x^2)\, dy = 0$.

11. a) Form the differential equation of the family of curves $y = a\sin(\omega t + b)$, where a and b are arbitrary constants.

b) In the Mean Value Theorem

$f(a+h) = f(a) + hf'(a+\theta h)$, if $a = 1$, $h = 3$ and $f(x) = \sqrt{x}$, find θ.

c) Solve $5^{x+1} + 5^{2-x} = 5^3 + 1$.

12. a) If $y = (x^2 - 1)^n$, then show that

$$(x^2 - 1)\, y_{n+2} + 2x\, y_{n+1} - n(n+1)\, y_n = 0.$$

b) If $f(x) = 2 - x$, when $1 \le x \le 2$

$= x - \frac{1}{2}x^2$, when $x > 2$, prove that $f(x)$ is continuous at $x = 2$.

c) Show that the minimum value of $4e^{2x} + 9e^{-2x}$ is 12.