CS/B.PHARM(O+N)/SEM-2/M-203/2010

2010

ADVANCED MATHEMATICS & ENGINEERING MECHANICS

Time Allotted : 3 Hours   Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Graph sheet(s) will be provided by the Institution

GROUP – A

(Multiple Choice Type Questions)

1.   Choose the correct alternatives for any *ten* of the following :

$$10 \times 1 = 10$$

i)   If A.M. of 9, $x - 2$, $x + 3$, 10 is 6, then the value of x is

a)   0   b)   2

c)   5   d)   none of these.

ii)   The events A and B are said to be mutually exclusive if

a) $P (A + B) = 1$ b) $P (AB) = P (A) P (B)$

c) $P (AB) = 0$ d)   none of these.

iii)   If $P (A) = 3/8$, $P (B) = 5/8$ and $P (A \cap B) = 1/4$, then $P (A / B)$ is

a)   2/5   b)   1/5

c)   2/3   d)   1/3.

iv) If two events A and B are independent, then

   a) A^C and B are independent

   b) A and B^C are independent

   c) A^C and B^C are independent

   d) all are true.

v) Two letters are drawn at random from the word HOME. Then the probability that both the letters are vowel is

   a) 1/3   b) 2/3

   c) 1/6   d) 1.

vi) Given that :

   Class limit :   10 – 20   20 – 30   30 – 40

   Frequency :   2   5   3

   Then the lower class boundary (L.C.B.) of 20 – 30 is

   a) 19·5   b) 20

   c) 20·5   d) none of these.

vii) The correlation coefficient lies between

   a) – 1 to 1   b) 1 to 2

   c) – 1 to 0   d) 0 to 1.

viii) The G.M. of 3, 12 and 48 is

   a) 6   b) 9

   c) 12   d) none of these.

ix) The mode of the following data

3, 2, 4, 9, 2, 8, 3, 2 is

a) 3   b) 2

c) 7   d) 9.

x) The probability mass function $f(x)$ must satisfy the two conditions :

a) $f(x) \geq 1$ and $\Sigma f(x) = 1$

b) $f(x) \geq 0$ and $\Sigma f(x) = 0$

c) $f(x) \geq 1$ and $\Sigma f(x) = 0$

d) $f(x) \geq 0$ and $\Sigma f(x) = 1$.

xi) $P(A + B + C)$ is equal to

a) $P(A) + P(B) + P(C)$

b) $P(A) + P(B) - P(C)$

c) $P(A)P(B)P(C)$

d) $P(A) + P(B) + P(C) - P(AB) - P(AC)$
$- P(BC) + P(ABC)$.

xii) The inverse Laplace Transform of $1/S(S^2 + 1)$ is

a) $1 + \cos t$ b) $1 - \cos t$

c) $1 + \sin t$ d) $1 - \sin t$.

xiii) If $L^{-1}\{f(s)\} = F(t)$, then $L^{-1}\{f(s - \pi/3)\}$ is

a) $(\pi/3)F(t)$ b) $F(t\pi/3)$

c) $e^{(\pi/3)t}f(t)$ d) $F(t - \pi/3)$.

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xiv) If C is a constant, then $L\,(\,C\,)$ is

    a) C b) C/S

    c) C/S^2 d)   None of these.

xv) If two mutually perpendicular forces P and Q act simultaneously at a point, then the resultant force is

    a) $P + Q$ b) $\sqrt{P^2 + Q^2 - PQ}$

    c) $\sqrt{P^2 + Q^2}$ d)   none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Test whether the events A and B are independent if $P\,(\,A\,) = 0\cdot4$, $P\,(\,B\,) = 0\cdot3$ and $P\,(\,A \cup B\,) = 0\cdot6$.

3. Evaluate $L\,\{\,t\,\sin bt\,\}$.

4. The equations of two regression lines between two variables x and y are expressed as

    $2x - 3y = 0$ and $4x - 5y - 8 = 0$.

Find $\bar{x}$, $\bar{y}$ and the coefficient of correlation between x and y.

5. A box contains 5 red and 10 white balls. 2 balls are drawn at random from the box one after another without replacement. Find the probability that both the balls drawn are white.

6. Find the magnitude of the two forces such that if they act at right angles, their resultant is $10^{\frac{1}{2}}$ N. But if they act at 60°, their resultant is $13^{1/2}$ N.

GROUP – C

(**Long Answer Type Questions**)

Answer any *three* of the following.   3 × 15 = 45

7.   a)   An experiment consists of throwing a die 5 times and noting the number of 6's.

The experiment was repeated 200 times with the following results :

Number of 6's :	0	1	2	3	4	5
Frequency :	58	86	40	14	2	0

Find the mean, median, mode and standard deviation for the above data.   10

b)   Draw a histogram of the following data :   5

Wages (Rs) :	50-59	60-69	70-79	80-89	90-99
No. of workers :	5	7	15	13	10

8.   a)   Find the centre of gravity of the homogeneous area bounded by the parabola $y^2 = 4ax$, the x axis and the ordinate $x = h$.   7

b)   A stone dropped into an emply pit of depth h is heard to strike the bottom after t sec. Prove that $2 h \{ 1 + (gt / v) \} = gt^2$, where v is velocity of sound supposed to be so large compared to h that $(h / v)^2$ can be neglected.   8

9.   The probability density function of a random variable x is

$f (x) = k (x - 9)$, $1 \leq x \leq 10$.

Determine —

a)   the value of the constant k 4

b)   the distribution function $F (x)$ 4

c) P **Error!**$\leq X \leq$ **Error!**) .   7
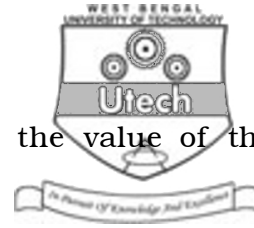
10. a) From the following result, calculate the value of the correlation coefficient :

$$\sum_{i=1}^{8} (X_i - \bar{X})^2 = 32, \quad \sum_{i=1}^{8} (Y_i - \bar{Y})^2 = 72 \text{ and}$$

$$\sum_{i=1}^{8} (X_i - \bar{X})(Y_i - \bar{Y} = 43. \qquad 4$$

b) If μ'_2 and μ_2 are the second moment about an arbitrary point a and about $\bar{x}$ respectively, show that $\mu'_2 = \mu_2 + d^2$, where $d = \bar{x} - a$. 4

c) The equations of two lines of regression are $2x + 3y - 8 = 0$ and $x + 2y - 5 = 0$. Identify the regression equation of y on x and x on y. Also find the ratio of the S.D. of x and y. 7

11. a) In an examination 30% of the students failed in Physics, 25% in Mathematics and 12% in both Physics and Mathematics. A student is selected at random. Find the probability that —

i) the student has failed in Physics, if it is known that he has failed in Mathematics.

ii) the student has failed in at least one of the two subjects.

iii) the student has passed in at least one of the two subjects.

iv) the student has passed in Mathematics if he failed in Physics. 8

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b) Find the Mean Deviation of the following frequency distribution : 7

Class-interval :	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
No. of workers :	8	10	12	9	5

12. a) Calculate the first 3 central moments and hence find the measure of skewness for the following data : 4

1, 3, 5, 7.

b) Find the coefficient of variation for the following data : 7

Class-interval :	4-6	6-8	8-10	10-12	12-14	14-16
Frequency :	13	10	9	5	8	5

c) Find out the missing frequencies from the following data, given that A.M. = 67·45 inches :

Height (inches) :	60-62	63-65	66-68	69-71	72-74	Total
No. of students :	5	18	f_3	f_4	8	100

4

13. a) Use Laplace Transforms to solve the following system of equations :

$$\frac{dx}{dt} + \frac{dy}{dt} + x + y = 1$$

$$\frac{dy}{dt} - 2x - y = 0$$

where $x(0) = 0$, $y(0) = 1$. 7

b) Evaluate $L^{-1}\left\{\dfrac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$. 4

c) If $L\{F(t)\} = f(s)$, prove that

$$L\{F(at)\} = \frac{1}{a}\, f\left(\frac{s}{a}\right). \qquad 4$$

14. a) Two forces $P + Q$ and $P - Q$ make an angle 2α with one another, and their resultant makes an angle θ with the bisector of the angle between them.

Show that $P \tan \theta = Q \tan \alpha$. 7

b) If a transversal cuts the lines of action OA_1 OA_2 ...... , OA_n of the forces P_1 , P_2 , P_n which are in equilibrium, at the points A_1 , A_2 , ..., A_n , then prove that

$$\frac{P_1}{OA_1} + \frac{P_2}{OA_2} + ... + \frac{P_n}{OA_n} = 0.$$ 4

c) A particle thrown vertically upwards takes t seconds to rise to a height h and subsequently takes $t^{\,l}$ seconds to reach the ground again. Prove that $h = \frac{1}{2} g\, t\, t^{\,l}$. 4

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