Name :
Roll No. :


Invigilator's Signature : $\qquad$

CS / B.PHARM (NEW) / SEM-2 / M-203 / 2011 2011

## ADVANCED MATHEMATICS \& ENGINEERING MECHANICS

Time Allotted : 3 Hours
Full Marks : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

## GROUP - A

## ( Multiple Choice Type Questions )

1. Choose the correct alternatives for any ten of the following :

$$
10 \times 1=10
$$

i) Rounding off 2.46289 correct to 3 places of decimals is
a) $2 \cdot 462$
b) 2.46
c) 2.463
d) none of these.
ii) If $P(A \cup B)=P(A)+P(B)-P(A \cap B)$ then the events $A$ and $B$ are
a) mutually exclusive
b) independent
c) both mutually exclusive and independent
d) none of these.
iii) If $P(A)=\frac{1}{2}, P(B)=\frac{1}{3}$ and $P(A B)=\frac{1}{4}$ then $R(A+B)$ is
a) $\frac{1}{24}$
b) $\frac{7}{12}$
c) $\frac{1}{8}$
d) none of these.
iv) If the event $A$ implies event $B$ then
a) $\quad P(A) \geq P(B)$
b) $\quad P(A) \leq P(B)$
c) $\quad P(A)=1-P(B)$
d) none of these.
v) In coefficient of variation $=\frac{k}{\text { Mean }} \times 100 \%$ where $k$ is
a) Mode
b) Standard Deviation
c) Variance
d) Median.
vi) Correlation coefficient lies between
a) 0 to 1
b) 1 to 2
c) -1 to 0
d) -1 to 1 .
vii) If $r=0 \cdot 5, \operatorname{cov}(x, y)=10$ and $\sigma_{y}=5$ then the value of $\sigma_{x}$ is
a) 10
b) 5
c) $0 \cdot 10$
d) 0.5
viii) Let $x$ be a Poisson random variable such that $2 p(x=0)=p(x=2)$. Then standard deviation of $x$ is
a) 4
b) 2
c) $-\sqrt{2}$
d) $\sqrt{2}$.
ix) Laplace transform of $(5 t-a)$ is
a) $\frac{5-a s}{s^{2}}$
b) $\frac{a s-5}{25}$
c) $\frac{5-a s}{s}$
d) none of these.
x) $\quad L\left(t e^{2 t}\right)$ is equal to
a) $\frac{1}{(s-2)}$
b) $\frac{2}{(S-2)^{2}}$
c) $\frac{1}{(s-2)^{2}}$
d) $\frac{21}{\mathrm{~s}^{2}}$.
xi) $\quad L^{-1}\left\{\frac{1}{s(s+4)}\right\}$ is equal to
a) $\frac{1}{4}\left(1-e^{-2 t}\right)$
b) $\frac{1}{2}\left(1-e^{-4 t}\right)$
c) $\left(1-e^{-2 t}\right)$
d) none of these.
xii) If two forces 100 N and 150 N are acting simultaneously at a point and if the angle between them is $45^{\circ}$, then the resultant of these two forces is
a) 232 N
b) 230 N
c) $\quad 175 \mathrm{~N}$
d) 200 N .
xiii) A particle is thrown vertically upwards with velocity $u$ then the greatest height attained by the particle is
a) $u / g$
b) $u^{2} / 2 g$
c) $(u / g)^{2}$
d) none of these.

2. Calculate the mode from the following data:

| class : | $20-24$ | $25-29$ | $30-34$ | $35-39$ | $40-44$ | $45-49$ | $50-54$ | $55-59$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| frequency: | 3 | 5 | 10 | 20 | 12 | 6 | 3 | 1 |

3. A random variable $x$ has the following probability density
function : $f(x)=\left\{\begin{array}{cc}2 x & 0<x<1 \\ 0 & , \text { otherwise }\end{array}\right.$

Find $P\left(\frac{1}{4}<x<\frac{1}{2}\right)$.
4. Find $L^{-1}\left(\frac{s-1}{(s-2)^{2}+1}\right)$.
5. Find the centre of gravity of the solid formed by the revolution about the $x$-axis, of the parabola $y^{2}=4 a x$ bounded by the ordinate $x=h$.
6. If a bomb, dropped from an aeroplane rising vertically with uniform velocity, reaches the ground in 5 seconds, find the height of the aeroplane when the bomb reaches the ground.


Answer any three of the following. $\quad 3 \times 15=45$
7. a) In a bolt factory the machines $M_{1}, M_{2}, M_{3}$ manufacture respectively $25 \%, 30 \%, 40 \%$ of the total product and their output $5 \%, 4 \%, 2 \%$ are defective respectively. One bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufacture by the machine $M_{3}$ ?
b) Find the coefficient of correlation for the following distribution :

| $x:$ | 10 | 14 | 18 | 22 | 26 | 30 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y:$ | 18 | 12 | 24 | 6 | 30 | 36 | $7+8$ |

8. a) Following table gives the frequency distribution of rainfall (in inches) in a certain locality for consecutive 106 days :

| Rainfall : | $0-5$ | $5-10$ | $10-15$ | $15-20$ | $20-30$ | $30-50$ | $50-70$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of days : | 5 | 10 | 25 | 20 | 18 | 20 | 8 |

Represent the distribution using a histogram.
b) Two urns contain respectively 3 white, 2 black balls and 2 white, 6 black balls. One ball is transferred from urn-I to urn-II and then one ball is drawn from the latter. It happens to be white. What is the probability that the transferred ball was black ? $8+7$

9. a) With the help of Laplace transform, solve $\frac{\mathrm{d} x}{\mathrm{~d} t}+3 y=2 x, \frac{\mathrm{~d} y}{\mathrm{~d} t}=y-2 x$ when, $x(0)=8$ and $y(0)=3$.
b) Apply the convolution theorem to find
$L^{-1}\left\{\frac{s}{\left(S^{2}+4\right)\left(s^{2}+9\right)}\right\}$.
10. a) Three forces $P, Q, R$ act along the sides of the triangle formed by the lines $x+y=1, y-x=1, y=2$. Prove that the equation of the line of action of their resultant is $P(x+y-1)+Q(y-x-1)-\sqrt{2} R(y-2)=0$. Also show that the magnitude of the resultant is $\left[P^{2}+Q^{2}+R^{2}-\sqrt{2} R(P+Q)\right]^{\frac{1}{2}}$
b) A uniform ladder is in equilibrium with one end resting on the ground and the other end against a vertical wall, if the ground and wall be both rough, the coefficient of friction being $\mu$ and $\mu^{\prime}$ respectively and if the ladder be on the point of slipping at both ends, show that the inclination of the ladder to the horizon is given by $\tan \theta=\frac{1-\mu \mu^{\prime}}{2 \mu}$.


11．a）Two forces $P$ and $Q$ acting parallel to the length and base of an inclined plane respectively would each of them singly support weight＇$W$＇on the plane then prove that $\frac{1}{P^{2}}-\frac{1}{Q^{2}}=\frac{1}{W^{2}}$
b）A stone falling from the top of a vertical tower has descended $x$ metre when another is let fall from a point $y$ metre below the top．If they fall from rest and reach the ground together，then show that the height of the tower is $(x+y)^{2} / 4 x$ metre． $7+8$

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