Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/MBA(OLD)/SEM-1 (FT & PT)/MB-105/2009-10

2009

QUANTITATIVE METHODS-I

Time Allotted : 3 Hours | Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) When three unbiased coins are tossed, probability of getting at least one head is

a) $\frac{1}{8}$ b) $\frac{3}{8}$

c) $\frac{7}{8}$ d) none of these.

ii) If in a determinant rows are changed into columns and vice versa, the value of the determinant

a) remains the same b) becomes zero

c) becomes one d) none of these.

iii) The x-coordinate of the point on the curve $y = x^2 - 2x + 3$ at which the tangent is perpendicular to the line $x + 3y + 3 = 0$ is

a) 7 / 2 b) 5 / 2

c) – 5 / 2 d) none of these.

iv) Which of the following functions is neither even nor odd ?

a) $\sin x$ b) $\cos x$

c) $\sin x + \cos x$ d) $x + \frac{1}{x}$.

v) Consider the word TIME. Form all possible subsets by using letters of this word. Then the number of proper subsets is

a) 16 b) 15

c) 12 d) 14.

vi) The no. of ways in which the letters of the word HOLIDAY can be arranged so that the vowels remain together is

a) 5040 b) 144

c) 720 d) 4320.

vii) Suppose the marginal cost function is given by $MC\ (q) = 4e^{2q}$. The fixed cost in 10. The total cost $TC\ (q)$ is

a) $2e^{2q} + 8$ b) $2e^{2q} + 10$

c) $4e^{2q} + 8$ d) $4e^{2q} + 10$.

viii) The functions f and g are defined by $f(x) = 2x$ and $g(x) = 1 / x, x \neq 0$. The value of $fg(x)$ is

a) $x / 2$b) $2 / x$

c) $2x^2$ d) $x^2 / 2$.

ix) If $P(A) = \frac{1}{4}$, $P(B) = \frac{3}{4}$ and $P(A \cap B) = \frac{3}{16}$, then the events A and B are

a) independent b) mutually exclusive

c) eqally likely d) none of these.

x) If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $ad - bc \neq 0$, then A^{-1} equals

a) $\begin{pmatrix} d & b \\ c & a \end{pmatrix}$

b) $\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

c) $\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

d) $\frac{1}{ad-bc}\begin{pmatrix} d & b \\ c & a \end{pmatrix}$.

xi) If $y = x^2 \ e^{mx}$, then $\frac{dy}{dx}$ equals

a) $x^2 + e^{mx}$

b) $mx^2 + e^{mx}$

c) $(mx + 2)\, e^{mx} . x$

d) $2xe^{mx}$.

xii) The centre of the circle passing through the origin and cutting off intercepts a and b on the X and Y axes is

a) $\left(\frac{a}{2}, \frac{b}{2}\right)$

b) $\left(\frac{-a}{2}, \frac{-b}{2}\right)$

c) (a, b)

d) $(a/2, -b/2)$.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Evaluate the following limit :

$$\lim_{x \to 1} \frac{\sqrt{x+8} - \sqrt{8x+1}}{\sqrt{5-x} - \sqrt{7x-3}} .$$

3. If $x^m \ y^n = (x + y)^{m+n}$, show that $\frac{dy}{dx} = \frac{y}{x}$.

4. Evaluate : $\int e^{mx} \cos x \, dx$.

5. A box contains 20 tickets of identical appearance, the tickets being numbered 1, 2, 3, ... 20. If 3 tickets are chosen at random, what is the probability that the numbers on the tickets drawn are in arithmetic progression ?

6. The line $y = mx$ and the curve $y = x^2 - 2x$ intersect at the origin O and meet again at a point A. If P is the mid-point of OA, find the equation of the locus of P as m varies.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \infty 15 = 45$

7. a) Show that the maximum value of the function $f(x) = x + \frac{1}{x}$ is less than its minimum value. Sketch the curve of this function, indicating the asymptotes, if any.

   b) The total cost function of a firm is given by $C(x) = \frac{1}{3}x^3 - 5x^2 + 28x + 10$, where x denotes the quantity of output. A tax at the rate of Rs. 2 per unit of output is imposed and the producer adds it to his costs. The market demand function is given by $P = 2350 - 5x$, where P denotes the unit price. Find the profit maximizing output and the corresponding price. 8 + 7

8. a) Find the equation of the straight line that passes through the interesection of the lines $3x + 4y = 17$ and $4x - 2y = 8$ and which is perpendicular to the line $7x + 5y = 12$.

   b) Find the equation of the circle which passes through the points (3, 4) and (3 – 6) and which has its centre on the straight line $2x + 3y = 3$.

   c) Show that the coordinates of the vertices of an equilateral traingle cannot be all rational. 5 + 5 + 5

9. a) Calculate the average value of $f(x) = 3 + |x|$ on the interval $[-2, 4]$.

b) Find the area bounded by the parabolas $y^2 = 4x$, $x^2 = 4y$ and the x–axis.

c) A function $f(x)$ is defined as follows :

$$f(x) = \frac{3}{2} - x, \text{ when } 0 < x < 1/2.$$

$$= \frac{1}{2}, \text{ when } x = 1/2$$

$$= 1/2 - x, \text{ when } 1/2 < x < 1.$$

Examine the continuity of the function at $x = 1/2$.

5 + 6 + 4

10. a) Find the value of x, in terms of a, b, c given that

$$\begin{bmatrix} x+a & b & c \\ c & x+b & a \\ a & b & x+c \end{bmatrix} = 0$$

b) State Cauchy's root test. Prove that the series

$$\frac{1}{3} + \left(\frac{2}{5}\right)^2 + \left(\frac{3}{7}\right)^3 + \ldots + \left(\frac{n}{2n+1}\right)^n + \ldots \text{ converges.}$$

c) If $A = \begin{bmatrix} 3 & 2 & 5 \\ 4 & 1 & 3 \\ 0 & 6 & 7 \end{bmatrix}$, express A as a sum of two matrices such that one is symmetric and the other skew-symmetric.

5 + 6 + 4

11. a) The letters of the word "THEORY" are permuted and then the words are arranged as in a dictionary. What is the rank of the word "THEORY" in that dictionary ?

b) A motorist plans a journey and event X is arrival at his destination in less than 3 hrs. He estinates the probability of dry weather, rain or snow to be 1/3, 1/2 and 1/6 respectively. The probabilities of event X in these conditions are 3/4, 2/5 and 1/10 respectively. What is the probability that the motorist completes his journey in 3 hours ? What is the probability that if he fails to arrive in less than 3 hours, there was a fall of snow ?

c) Evaluate : $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} \, dx$. 6 + 6 + 3