





iii) The number of distinct ways in which 7 distinct toys can be distributed among 3 children is

- a)  $3^7$
- b)  $7^3$
- c)  ${}^7C_3$
- d)  ${}^7P_3$ .

iv) If  $P(A) = \frac{3}{4}$ ,  $P(B) = \frac{1}{2}$  and  $P(A \leftrightarrow B) = \frac{3}{8}$ , then the events  $A$  and  $B$  are

- a) independent
- b) mutually exclusive
- c) equally likely
- d) none of these.

v) If  $y = xe^x$ , then  $\frac{dy}{dx}$  is

- a)  $x + e^x$
- b)  $e^x$
- c)  $x$
- d)  $(x + 1)e^x$ .

vi) If  $y = a \ln |x| + bx^2 + x$ , has its extreme values at  $x = -1$  and  $x = 2$ , then

- a)  $a = 2, b = -1$
- b)  $a = 2, b = -\frac{1}{2}$
- c)  $a = -2, b = \frac{1}{2}$
- d) none of these.

vii) What is the position of the point  $(-3, 3)$  with respect to the circle  $x^2 + y^2 - 4x - 6y = 3$ ?

- a) Inside the circle
- b) On the circle
- c) Outside the circle
- d) At the centre of the circle.





**GROUP – B**

**( Short Answer Type Questions )**

Answer any *three* questions. 3 × 5 = 15

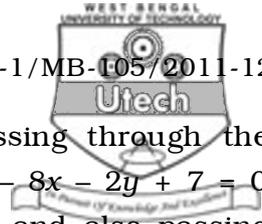
2. Evaluate the following limits :
  - a)  $\lim_{x \rightarrow 0} \frac{\sin x^\circ}{x}$
  - b)  $\lim_{x \rightarrow a} \frac{x \sin a - a \sin x}{x - a}$  . 2 + 3
3. If  $A = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$  and  $B = \begin{pmatrix} -1 & 0 \\ 5 & 1 \end{pmatrix}$  verify that  
 $(AB)^{-1} = B^{-1} A^{-1}$  .
4. a) If  $A = \{ x : 1 \leq x \leq 3 \}$ ,  $B = \{ x : 2 \leq x \leq 4 \}$ , find the sets  $A \approx B$ ,  $A \leftrightarrow B$  and  $A - B$ .  
 b) Find the domain of definition of the following function :  
 $f(x) = \frac{\sqrt{x-2}}{x^2-x}$  . 3 + 2
5. Find the number of ways in which a mixed doubles game can be arranged from among 9 married couples if no husband and wife play in the same game.
6. Find the area bounded by the curves  $y = x^2$  and  $y = x$ .

**GROUP – C**

**( Long Answer Type Questions )**

Answer any *three* questions. 3 × 15 = 45

7. a) The equations of two sides of a triangle  $PQR$  are  $x + 2y + 4 = 0$  and  $3x - 4y + 37 = 0$ . Given that  $\angle PQR = 90^\circ$  and that  $P$  is the point  $( 6, -5 )$ , find the coordinates of  $Q$ .



b) Find the equation of the circle passing through the intersection of the circles  $x^2 + y^2 - 8x - 2y + 7 = 0$  and  $x^2 + y^2 - 4x + 10y + 8 = 0$  and also passing through the point  $(3, -3)$ .

c) Test the convergence / divergence of the following series :

$$\frac{1}{3} + \left(\frac{2}{5}\right)^2 + \left(\frac{3}{7}\right)^3 + \dots + \left(\frac{n}{2n+1}\right)^n + \dots$$

5 + 5 + 5

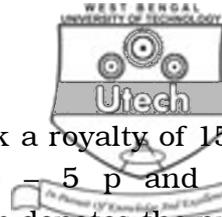
8. a) A man is known to speak truth 3 out of 4 times. He throws a dice and reports that it is a six. What is the probability that it is actually a six ?

b) A company manufactures soap, powder and toothpaste and wishes to know about its market in a particular area. A survey was conducted covering a population of 1899 persons. From the survey report the manager came to know that 30 persons had been using all the three products, 757 persons had been using soap, 574 persons powder, 132 persons soap and powder, 155 persons soap and toothpaste, 472 persons toothpaste and 47 persons toothpaste and powder. 398 persons did not use any of the products. After studying the report, the manager concluded that it was wrong. Discuss.

c) Show that

$$\Delta = \begin{vmatrix} b^2 + c^2 & ab & ac \\ ab & c^2 + a^2 & bc \\ ca & cb & a^2 + b^2 \end{vmatrix} = 4a^2 b^2 c^2 .$$

5 + 5 + 5



9. a) A publisher pays the author of a book a royalty of 15%. Demand for the book is  $x = 200 - 5p$  and the production cost is  $C = 10 + 2x + x^2$ .  $p$  denotes the price of the book. Find the optimal sales from both the author's and the publisher's perspective.

b) A student is preparing for exams in two subjects. He estimates that the grades he will obtain in each subject, as a function of the amount of time spent working on them are :

$$g_1 = 20 + 20\sqrt{t_1}$$

$$g_2 = -80 + 3t_2$$

where  $g_i$  is the grade in subject  $i$  and  $t_i$  is the number of hours per week spent in studying for subject  $i$ ,  $i = 1, 2$ . He wishes to maximize his grade average  $(g_1 + g_2) / 2$ . He cannot spend in total more than 60 hours studying in the week. Find the optimal values of  $t_1$  and  $t_2$  and discuss the characteristics of the solution.

c) Evaluate :

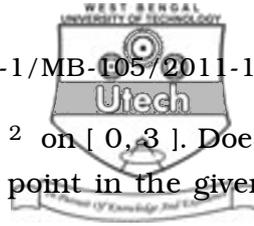
$$\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx.$$

6 + 6 + 3

10. a) Evaluate :

$$3x\sqrt{1 - 2x^2} dx.$$

b) An industry has its current level of production equal to 10,000 units. It is estimated that the rate of change of production  $p$  with respect to change in the amount of machine capacity  $x$  is given by  $(dp / dx) = 100 - 3\sqrt{x}$ . How much should production increase if the additional capacity available is 25 units ?



c) Find the average value of  $f(x) = 4 - x^2$  on  $[0, 3]$ . Does  $f$  actually take on this value at some point in the given domain ?

d) A function  $f(x)$  is defined as follows :

$$\begin{aligned} f(x) &= 2x - 1, \text{ if } x < 3 \\ &= a, \text{ if } x = 3 \\ &= b - x, \text{ if } x > 3. \end{aligned}$$

Find the values of  $a$  and  $b$ , if  $f(x)$  is continuous at  $x = 3$ .

$$3 + 5 + 4 + 3$$

11. a) A company manufactures three goods  $X$ ,  $Y$  and  $Z$ , each of which is made from three types of input  $A$ ,  $B$  and  $C$ . Each unit of  $X$  requires 1 unit of  $A$ , 7 units of  $B$  and 3 units of  $C$ . Each unit of  $Y$  requires 4 units of  $A$ , 3 units of  $B$  and 1 unit of  $C$ . One unit of  $Z$  requires 2 units of  $A$ , 4 units of  $B$  and 2 units of  $C$ . In a particular day's production the company uses up 105 units of  $A$ , 135 units of  $B$  and 55 units of  $C$ .

- i) Create a matrix equation to represent the usage of  $A$ ,  $B$  and  $C$  in the day's production of  $x$ ,  $y$  and  $z$  units of  $X$ ,  $Y$  and  $Z$  respectively.
- ii) Using matrix algebra, determine values of  $x$ ,  $y$  and  $z$ .

b) Find the total number of possible arrangements of 3 letters chosen from the word CALCULUS.

c) Find the set of values of  $x$  for which the series with  $n$ th term  $\frac{x^{2n}}{n}$  converges.

$$(2 + 5) + 4 + 4$$