	Uiteah
Name:	
Roll No.:	To Annual (NE Sampledge Staff Confident)
Invigilator's Signature :	

#### 2012

#### **OPERATIONS RESEARCH**

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

# GROUP – A ( Multiple Choice Type Questions )

1. Choose the correct alternatives for any *ten* of the following:

 $10 \times 1 = 10$ 

- i) A feasible solution in an LPP, in which at least (n-m) variables are zero and the vectors associated with the remaining variables are linearly independent is called
  - a) basic solution
- b) basic feasible solution
- c) non-basic solution
- d) degenerate solution.
- ii) In an assignment problem, the minimum number of lines covering all zeroes in the reduced cost matrix of order n, can be
  - a) at most n
- b) n + 1

c) n-1

d) m - n + 1.

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iii) The number of basic variables in a transportation problem of m sources and n destinations is at most

m + n + 1

b) m + n

m + n - 1c)

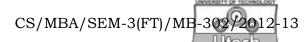
d) m - n + 1.

The basic feasible solutions of the system of equations iv)  $x_1 + x_2 + x_3 = 8$ ,  $3x_1 + 2x_2 = 18$  are

no basic solution b) (2, 6, 0), (6, 0, 2)

c) (1, 7, 0), (7, 1, 0) d) (2, 2, 0), (6, 6, 2).

- Which statement of the following is false in case of v) simulation?
  - Simulation can be done in case of probabilistic as a) well as deterministic models
  - b) Simulation uses random numbers
  - c) Simulation explains a complex system realistically
  - d) Simulation is used when we can develop an analytical solution of the problem.
- In Markov process the steady state refers to vi)
  - initial state a)
  - b) final state
  - state when probability is time dependent c)
  - d) state when probability is time independent.
- For Queuing model M/M/1:  $\infty$ /FIFO, which of the vii) following is not true?
  - Input follows Poisson distribution a)
  - b) Output process follows exponential distribution
  - c) Capacity is finite
  - It has only one service provider. d)



- viii) The real life examples of integer programming problem is
  - a) Knapsack problem
- b) Cutting plane
- c) Branch and bound
- d) none of these.
- ix) The degeneracy in transportation problem occurs when
  - a) all non-basic variables become zero
  - b) all basic variables become zero
  - c) some basic variables become zero
  - d) none of these.
- x) Charne's Big-M method is applied when
  - a) constraints are ≥
  - b) constraints are  $\leq$
  - c) constraints are =
  - d) artificial variable is used.
- xi) A queue-system will remain unchanged if
  - a)  $\rho < 1$

b)  $\rho = 1$ 

c)  $\rho > 1$ 

- d) none of these.
- xii) An assignment problem is
  - a) an LPP
  - b) an IPP
  - c) a transportation problem
  - d) all of these.





#### (Short Answer Type Questions)

Answer any three of the following

 $3 \times 5 = 15$ 

2. Find the basic solution to the following problem:

Maximize 
$$Z = x_1 + 3x_2 + 3x_3$$

subject to 
$$x_1 + 2x_2 + 3x_3 = 4$$

$$2x_1 + 3x_2 + 5x_3 = 7$$

Also find which of the basic solutions are

- a) Basic feasible
- b) Non-degenerate basic feasible
- c) Optimal basic feasible.
- 3. A firm is engaged in producing two products *A* & *B* which yield a unit profit of Rs. 400 and Rs. 300 respectively. Both of these require a certain raw material whose supply is limited to 300 kg per month. The raw material requirement is 1 kg per unit of each product. Further it is known that one unit of *A* requires 2 hours of processing while each unit of *B* needs 1 hour. The firm has a normal processing capacity of 400 hours per month. The marketing department has stated that according to the prevailing conditions and likely to continue in the foreseeable future, the maximum number of units of *A* & *B* which can be sold each month is 150 and

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350 respectively. The manager of the firm has set the following goals in order of their importance

Goal 1 : Avoid under-utilization of normal productive capacity

Goal 2: Sell the maximum number of units of products A & B. However, since the two products yield the profit in the ratio 400:300, the manager would give the weightage to the sale of the products in the same ratio.

Goal 3: Minimize overtime of processing capacity.

Formulate the above as a Goal programming problem.

4. Find the dual of the following problem:

Maximize 
$$Z = x_1 - 2x_2 + 3x_3$$

subject to 
$$3x_1 + 2x_2 - x_3 = 8$$

$$2x_1 - x_2 + 5x_3 \ge 3$$

$$x_1 + 2x_2 - 3x_3 \le 4$$

$$x_1, x_2 \ge 0, \ x_3$$
 , unrestricted in sign.

- 5. There is congestion on the platform of a railway station. The trains arrive at a rate of 30 trains per day. The average time spent in the station for a train is exponentially distributed with an average of 36 minutes.
  - a) What is the mean queue size?
  - b) The average time of a train that has to wait?
  - c) Probability that there are more than 5 trains in the queue.

6.		$D_1$	$D_2$	$D_3$	Availability
	$O_1$	9	10	12	n 10 Emilion 2nd Excland
	$O_2$	15	14	12	20
	$O_3$	16	16	15	30
	Demand	15	25	20	60

Write the corresponding LPP to minimize the cost of the transportation problem. Also find the IBFS by North-West Corner method.

# GROUP – C ( Long Answer Type Questions )

Answer any *three* of the following.  $3 \times 15 = 45$ 

7. a) Solve using Big-M method:

Maximize 
$$Z = 5x_1 + 8x_2$$
  
subject to  $3x_1 + 2x_2 \ge 3$   
 $x_1 + 4x_2 \ge 4$   
 $x_1 + x_2 \le 5$   
 $x_1, x_2 \ge 0$ 

b) A repairman is to be hired to repair machines which break down at a rate of 3 per hour. Non-productive time on any machine will cost the company Rs. 20 per hour. Two repairmen are available. One is slow-cheap who repairs at an average rate of 4/hour and demands Rs. 40/per hour. The other is fast-expensive who repairs at an average rate of 6/hour and demands Rs. 72/hour. Which repairman should be hired? 9 + 6

8. a) A manufacturer has distribution centres located at Agra, Allahabad and Kolkata. These centres have available 40, 20 & 40 units of products. His retail outlets require the following number of units:

A-25, B-10, C-20, D-30 and E-15. The transportation cost per unit in rupees between each centre and outlet is given in the following table:

	Retail outlets					
Distribution centres	A	В	С	D	E	
Agra	55	30	40	50	50	
Allahabad	35	30	100	45	60	
Kolkata	40	60	95	35	30	

Determine the optimal transportation cost.

b) A newspaper boy sells newspapers on buses during off peak hours. He can purchase at a concession rate of 50 paise and sell it for 60 paise. Any unsold copy is a dead loss. He has estimated the following probabilities for the number of copies demanded:

Number of copies	20	21	22	23	24	25
Probability	0.1	0.15	0.25	0.3	0.12	0.08

Prepare a pay-off table to find out how many copies should be ordered so that expected profit is maximized.

9. a) Records of 100 truck loads of finished jobs arriving in a department's check-out area show the following: Checking out takes 5 minutes and checker takes care of only one truck at a time.

Truck inter- arrival time (minutes)	1	2	3	4	5	6	7	8	9	10
Frequency	1	4	7	17	31	23	7	5	3	2

As soon as the trucks are checked out, the truck drivers take them to the next department. Using Monte Carlo simulation, determine the average waiting time of trucks before service, assuming the checking starts at 10 a.m. Random numbers given are: 12, 81, 36, 82, 21, 74, 90, 55, 79, 70, 14, 59, 62, 57, 15, 18, 74, 11, 41, 29.

- b) A self-service store employs one cashier at its counter. An average of nine customers arrive every 5 minutes while the cashier can serve 10 customers in 5 minutes. Assuming Poisson distribution for arrival rate and exponential distribution for service rate, find
  - i) average number of customers in the system
  - ii) average number of customers in the queue or average queue-length
  - iii) average time a customer spends in the system
  - iv) average time a customer waits before being served.

8 + 7

10. a) Durham Designs manufactures home furnishings for department stores. Planning is underway for the production of items in the 'Wildflower' fabric pattern during the next production period:

	Bedspread	Curtains	Dust Ruffle
Fabric required(yds)	7	4	9
Time required(hrs)	1.5	2	0.5
Packaging material	3	2	1
Profit (Rs.)	12	10	8

Inventory of the Wildflower fabric is 3000 yards. Five hundred hours of production time have been scheduled. Four hundred units of packaging material are available. Each of these values can be adjusted through overtime or extra purchases.

Durham would like to achieve a profit of Rs. 3,200, avoid purchasing more fabric or packaging material and use all of the hours scheduled. Give the goal programming model.

b) Develop the Branch and Bound tree for the following problem:

Maximize 
$$Z = 3x_1 + 2x_2$$

subject to 
$$2x_1 + 5x_2 \le 9$$

$$4x_1 + 2x_2 \le 9$$

$$x_1, x_2 \ge 0$$
 and integer.

6 + 9

11. a) The following table summarizes the activities of three companies in terms of gains and losses during a given period of one month:

Company	No. of customers on 1/5/12	Gains from			Losses to			No. of customers on 1/6/12
A	500	0	10	20	0	15	10	505
В	500	15	0	5	10	0	0	510
C	500	10	0	0	20	5	0	485

Prepare the transition matrix. If the exchanges remain unchanged, what will the market share on 1/7/12? Also find the steady state probability.

b) Mr. Iyer is a salesman with Delite Manufacturing Company. He stays in city A and wants to visit cities B, C, D & E and come back to the city A at the end of the day. He wants to travel minimum distance and does not want to travel to other city twice. The following table gives the inter-city distance. Find the sequence of cities and also find the distance travelled.

Enama Oitaa	To city					
From City	A	В	C	D	E	
A	_	4	7	3	4	
В	4	_	6	3	4	
C	7	6	_	7	5	
D	3	3	7	_	7	
E	4	4	5	7	_	

(2+3+4)+6

12. a) An individual was interested in determining which of two stocks to invest in, Central Computing Company (CCC) or Software Research Inc. (SRI). The criteria thought to be most relevant in making the decision are the potential yield of the stock and the risk associated with the investment. The pairwise comparison matrices for this problem are:

Criterion

	Yield	Risk
Yield	1	2
Risk	$\frac{1}{2}$	1

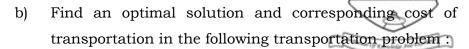
Yield

	CCC	SRI
CCC	1	3
SRI	$\frac{1}{3}$	1

Risk

	CCC	SRI
CCC	1	$\frac{1}{2}$
SRI	2	1

- i) Compute the priorities for each pairwise comparison matrix.
- ii) Determine the overall priority for the two investments, CCC and SRI. Which investment is preferred based on yield and risk?



	$D_1$	$D_2^{}$	$D_3$	$D_4$	
$O_1$	1	2	- 2	3	70
$O_2$	2	4	0	1	38
$O_3$	1	2	- 2	5	32
	40	28	30	42	_

8 + 7

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