Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/MBA (NEW)/SEM-4 (PT)/MB-302/2011

2011

OPERATIONS RESEARCH

Time Allotted : 3 Hours Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Graph sheet(s) will be supplied by the Institution.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) If two constraints do not intersect in the positive quadrant of the graph, then

a) one of the constraints is redundant

b) the solution is infeasible

c) the solution is unbounded

d) has a unique solution.

ii) Given a system of 'm' simultaneous linear equations in 'n' unknowns ($m < n$), the number of basic variables will be

a) m b) n

c) $n - m$ d) $m + n$.

iii) The role of artificial variables in simplex method is

a) to aid in finding initial basic feasible solution

b) to start phases of simplex method

c) to find shadow prices from the final simplex table

d) none of these.

iv) If optimum solution is degenerate, then

a) the solution is infeasible

b) there are alternative optimum solutions

c) the solution is of no use to the decision maker

d) the solution is unbounded.

v) If dual has an unbounded solution, primal has

a) an unbounded solution

b) an infeasible solution

c) a feasible solution

d) basic feasible solution.

vi) Dual simplex method is applicable to those L. P. P's that start with

a) an infeasible solution

b) an infeasible but optimum solution

c) a feasible solution

d) a feasible and optimum solution.

vii) The solution of transportation problem with 'm' sources and 'n' destinations is non-degenerate, if the number of allocations are

a) $m + n - 1$ b) $m + n + 1$

c) $m + n$ d) $m.n$.

viii) In an assignment problem involving four workers and three jobs, total number of assignments possible is

a) 4 b) 3

c) 7 d) 12.

ix) When there are more than one servers, customers behaviour in which he moves from one queue to another is

a) Balking b) Jockeying

c) Reneging d) Alternating.

x) Multiple servers may be

a) in parallel

b) in series

c) combination of parallel and series

d) all of these.

xi) Markov analysis is useful in

a) calculating transition probabilities at some future time

b) predicting the state of the system at some future time

c) both of these

d) none of these.

xii) In Markov analysis, the state probabilities must

a) sum to one b) be less than one

c) be greater than one d) all of these.

GROUP – B
(Short Answer Type Questions)

Answer any *three* of the following. 3 × 5 = 15

2. At a railway station, only one train is handled at a time. The railway yard is sufficient only for two trains to wait while other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them on an average of 12 per hour. Assuming Poisson arrivals and exponential service distribution, find the steady state probabilities for the various numbers of trains in the system. Also find the average number of trains coming into the yard.

3. Find initial basic feasible solution of the following transportation problem by Matrix Minima Method and check whether the problem is degenerate or not. If it is degenerate, to which cell/cells the arbitrary allocation/allocations will be made ? 3 + 2

	P	*Q*	*R*	*S*	Capacity
A	1	2	3	4	6
B	4	3	2	0	8
C	0	2	2	1	10
Demand	4	6	8	6	

4. A farmer wants to decide which of the three crops he should plant on his 100 acre farm. The profit from each is dependent on the rainfall during the growing season. The farmer has categorized the amount of rainfall as high, medium and low. His estimated profit for each is shown below :

Rainfall	Estimated conditional profit (in Rupees)		
	Crop A	Crop B	Crop C
High	5000	3500	5000
Medium	4500	4500	5000
Low	2000	5000	4000

If the farmer wishes to plant only one crop, decide which should be his 'the best crop' using 'Maximax criteria' ?

5. With reference to Queuing Theory, what do you mean by ($M/M/1$) : ($\infty/FIFO$) ? What do you mean by "transient state" and "steady" state of queuing model ?

GROUP – C
(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

6. a) Find all integer solution of the following L. P. P :

Maximize $Z = 2x + 3y$

subject to :

$3x + 4y \leq 12$

$x + 3y \leq 6$

$x, y \geq 0$. 8

b) A firm having two sources *A* and *B* wishes to ship its products to two destinations *P* and *Q*. The numbers of units available at *A* and *B* are 5 and 25 respectively. The products demanded at *P* and *Q* are 20 and 10 units respectively. The firm instead of shipping directly from sources to destinations, decides to investigate the possibility of transhipment. The unit transportation costs (in rupees) are given in the following table. Find the optimal shipping schedule.

		Source		Destination		Availability
		A	*B*	*P*	*Q*	
Source	*A*	0	2	3	4	5
	B	2	0	2	4	25
Destination	*P*	3	2	0	1	—
	Q	4	4	1	0	—
Demand		—	—	20	10	

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7. a) A company has a single service station which has the following characteristics :

The mean arrival rate of customers and the mean service time are 6·2 minutes and 5·5 minutes respectively. The time between an arrival and its services varies from one minute to seven minutes. The arrival and service time distributions are given below :

Time (in minute)	Arrival (Probability)	Service (Probability)
1 – 2	0·05	0·10
2 – 3	0·20	0·20
3 – 4	0·35	0·40
4 – 5	0·25	0·20
5 – 6	0·10	0·10
6 – 7	0·05	—

The queuing process starts at 11 A.M. and closes at 12 P.M. An arrival moves immediately into the service facility if it is empty. On the other hand, if the service station is busy, the arrival will wait in the queue. Customers are served on the first come, first served basis. If the clerk's wages are Rs. 6 per hour and the customer's waiting line cost Rs. 5 per hour, would it be economical for the manager to engage the second clerk ? Use Monte Carlo simulation technique. 10

b) Formulate the following Diet problem as L. P. P : A patient needs daily 5 mg, 20 mg & 15 mg of vitamins *A*, *B* and *C* respectively. The vitamins available from a mango, an orange and an apple are 0·5 mg of *A*, 1 mg of *B*, 1 mg of *C*; 2 mg of *B*, 3 mg of *C*; 0·5 mg of *A*, 3 mg of *B* and 1 mg of *C* respectively. If the cost of a mango, an orange and an apple be Re. 0·50, Re. 0·25 and Re. 0·40 respectively, find the minimum cost of collecting the fruits so that daily requirement of the patient be met. Formulate it. 5

8. a) In a small town with three advocates *X*, *Y* & *Z* each advocate knows that some clients switch back & forth, depending on which advocate is available at the time the client needs one. There are no new clients in the current legal market; however, none of the old clients are leaving the area. During a slack period, the three advocates collected data which identified the number of clients each advocate had seen during the preceding year. Tables given below summarize the results of this study and the manner in which clients were gained or lost respectively. Construct the state-transition matrix that describes the problem at hand. 8

DATA SUMMARY — CLIENT ACTIVITY

Advocate	Clients as of January 1, 2005	Change during year		Clients as of January 1, 2006
		Gain	Loss	
X	400	75	50	425
Y	500	50	150	400
Z	500	100	25	575

GAIN — LOSS SUMMARY

Advocate	**Clients as of January 1, 2005**	**Gains**			**Loss**			**Clients as of January 1, 2006**
		From *X*	**From *Y***	**From *Z***	**To *X***	**To *Y***	**To *Z***	
X	**400**	**0**	**50**	**25**	**0**	**50**	**0**	**425**
Y	**500**	**50**	**0**	**0**	**50**	**0**	**100**	**400**
Z	**500**	**0**	**100**	**0**	**25**	**0**	**0**	**575**

b) Given the following L. P. P :

Maximize $Z = x_1 + 4x_2 - 2x_3 + 3x_4 + x_5$

subject to :

$$x_1 - 3x_2 + x_3 + 2x_4 + 6x_5 \leq 3$$

$$2x_1 + x_2 + 3x_4 + 2x_5 \leq 6$$

$$4x_1 + x_2 - x_4 + x_5 \leq 2$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Find the ranges over which c_1, c_3, c_4, b_3 can be changed so that the opportunity of the current solution remains undisturbed. (Assume that when c_1 changes, all other quantities remain unchanged). Apply Sensitivity Analysis for the problem. 7

9. a) *XYZ* Airline operating 7 days a week has given the following time table. Crews must have a minimum layover of 5 hours between flights. Obtain the paring flights that minimize layover time away from home. For any given pairing the crew will be based at the city results in the smaller layover : 8

Chennai – Mumbai			Mumbai – Chennai		
Flight Number	Depart	Arrive	Flight Number	Depart	Arrive
1	6 AM	8 AM	101	8 AM	10 AM
2	8 AM	10 AM	102	9 AM	11 AM
3	2 PM	4 PM	103	2 PM	4 PM
4	8 PM	10 PM	104	7 PM	9 PM

b) Arrivals at a telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 3 minutes.

i) What is the probability that a person arriving at the booth will have to wait ?

ii) What is the average length of the queue that forms from time to time ?

iii) The telephone department will instal a second booth when convinced that an arrival would expect to have to wait at least three minutes for the phone,. By how much must the flow of arrivals be increased in order to justify second booth ? 7

10. a) Give the mathematical formulation of transportation problem. 5

b) i) Find the initial basic feasible solution of the following transportation problem using Vogel's Method. 5

Warehouse → Factory ↓	*P*	*Q*	*R*	*S*	Factory Capacity
A	19	30	50	10	7
B	70	30	40	60	9
C	40	8	70	20	18
Warehouse Requirement	5	8	7	14	

ii) Is this solution an optimal solution ? If not, obtain the optimal solution. 5