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Paper Code : M(MCA)-101

DISCRETE MATHEMATICAL STRUCTURE

Time Allotted : 3 Hours | *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) Solution of the recurrence relation $a_n = 2a_{n-1} + 1$ with $a_0 = 0$ is

   a) $1 - 2^n$ b) $2^n - 2$

   c) $2^{n-1} - 1$ d) $2^n - 1$.

   ii) If the truth value of p and q are T and F respectively, then the truth value of $\sim p \rightarrow \sim q$ is

   a) T b) F

   c) both T and F d) none of these.

iii) The mapping $f : N \to N$ defined by $f(n) = \left[\frac{n+1}{2}\right]$, $n \in N$ is

a) injective b) surjective

c) bijective d) none of these.

iv) Choose the correct statement :

a) Path is an open walk

b) Every walk is trail

c) Every trail is path

d) A vertex cannot appear twice in a walk.

v) The type-2 Grammar in relation to the automata theory is known as

a) context sensitive grammar

b) regular grammar

c) context free grammar

d) none of these.

vi) How many ways can the letters of the word 'MADAM' be arranged ?

a) 30 b) 24

c) 120 d) 60.

vii) Which one is a tautology ?

a) $p \vee q$ b) $p \vee \sim q$

c) $p \vee \sim p$ d) $p \vee F$.

viii) The type of the grammar, which consists of which of the following productions ?

$s \to aA,\ A \to aAB,\ B \to b,\ A \to a$

a) Type -0 b) Type-1

c) Type-2 d) Type-3.

ix) The generating function of the sequence $\{ 0, 0, 1, 1, 1, 1, 1, \ldots\ldots \}$ is

a) $x^2(1+x)^{-1}$ b) $x(1+x)^{-2}$

c) $x(1-x)^{-1}$ d) $x^2(1-x)^{-1}$.

x) If a tree has 10 vertices then number of its edges is

a) 8 b) 11

c) 10 d) none of these.

xi) Minimal spanning tree in a graph is

a) unique b) not unique

c) more than three d) exactly three.

xii) Let L be a language given by $L = \{ a^n b^n : n \geq 0 \}$ then L^2 is equal to

a) $\{ a^n b^n a^m b^m : n \geq 0 \ \ m \geq 0 \}$

b) $\{ a^n b^n : n \geq 0 \}$

c) $\{ a^n b^n a^m b^m : n \geq 0 \}$

d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Show that $f : R \to R$ defined by $f(x) = x^3 - 2$ is bijective. Find f^{-1}.

3. Prove by mathematical induction :

$$3 + 33 + 333 + \ldots\ldots\ldots + 33 \ldots\ldots\ldots 3 = \frac{10^{n+1} - 9n - 10}{27}.$$

4. Solve the recurrence relation

$a_n - 6a_{n-1} + 8a_{n-2} = n - 1,\ n \geq 2,\ a_0 = 1,\ a_1 = 3.$

5. A relation on the set $A = \{ 4, 6, 8, 10 \}$ whose matrix representation is given by

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Find the transitive closure of R.

6. Show that in a Binary tree with n vertices has $\frac{n+1}{2}$ pendant vertices.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) What is complemented lattice ? Give an example.

   b) Show that the set of all positive divisors of 72 forms a poset under the relation $\leq$ defined as $a \leq b$ if a is a divisor of b. Draw its Hasse diagram.

c) Let $\sum = (a, b)$, write the grammar for the language $L = \{ a^n b a^n : n \geq 0 \}$. 4 + 6 + 5

8. a) Obtain the DNF of $(P \wedge Q) \vee [\ \neg(P \rightarrow Q)\ ]$.

b) Convert the following arguments in the predicates and check whether their conclusions are valid :

All doctors are brilliant.

All brilliant are laborious.

Rajesh is not labourious.

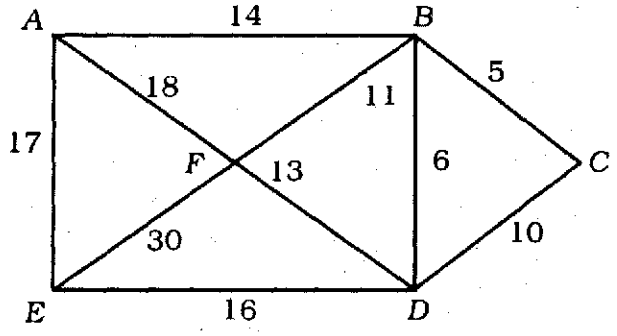
Therefore, Rajesh is not doctor.

c) Construct a DFA from the NFA :

State	Input a = 0	Input a = 1
A	B	B, C
B	A, C	.....
C	A	B, C

5 + 5 + 5

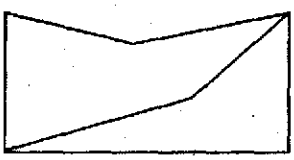
9. a) Define minimal spanning tree of a graph with an example. Apply Prim's Algorithm to the following weighted graph and find the minimum weight.



b) How many solutions does the equation $x_1 + x_2 + x_3 + x_4 = 30$ have if $x_1 \geq 2$, $x_2 \geq 4$, $x_3 \geq 5$ and $x_4 \geq 6$ and all are integers.

c) Test whether the following two graphs G_1 and G_2 are isomorphic to each other or not.

G_1



G_2

8 + 4 + 3

10. a) Use mathematical induction to prove that $n^3 + 2n$ is divisible by 3.

b) Show that the mapping $f : R - \{\sqrt{2}\} \to R$ defined by $f(x) = \frac{x}{x^2 - 2}$, $x \neq \sqrt{2}$ is surjetive but not injective.

c) Construct a minimum state automata equivalent to the finite automata given below :

Present state	Next state	
	Input a = 0	Input a = 1
$\to q_0$	q_1	q_5
q_1	q_5	q_2
	q_0	q_2
q_3	q_2	q_4
q_4	q_5	q_5
q_5	q_2	q_4

11. a) Prove that if there is one and only path between every pair of vertices in a graph G, then G is a tree.

b) If $U = \{ 2, 3, 4, \ldots\ldots 8 \}$ and the two fuzzy sets are $F_1 = \{ (2, 0{\cdot}7), (5, 0{\cdot}7), (6, 1) \}$,

$F_2 = \{ (2, 0{\cdot}6), (5, 0{\cdot}8), (7, 1) (8, 0{\cdot}1) \}$.

Find $\overline{B}$, $\overline{A} \cap \overline{B}$, $A \cup B$.

c) Apply Kruskal's Algorithm to find the minimum spanning tree of the following graph :