MM-101

DISCRETE MATHEMATICAL STRUCTURE

Time Allotted: 3 Hours — Full Marks: 70

The questions are of equal value.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP A
(Multiple Choice Type Questions)

1. Answer any *ten* questions. $10\times1 = 10$

(i) The number of arrangements of 25 objects where 7 are of the first kind, 2 are of the second kind, 3 are of the third kind and 4 are of the fourth kind is given by

(A) $\frac{25!}{7!2!3!4!}$ (B) $\frac{25!}{7!2!}$ (C) $\frac{25!}{3!4!}$ (D) none of these

(ii) If A, B and C are any three arbitrary sets, then $A-(B\cap C)$ is

(A) $(A-B)\cup(A-C)$ (B) $(A-B)\cap(A-C)$
(C) $(A-B)\cap(C-A)$ (D) $(B-A)\cup(A-C)$

(iii) If A and B are two fuzzy sets given by A = {(1, 0.1), (3, 0.4), (5, 0.2), (7, 0.8)} and B = {(1, 0.3), (3, 0.2), (5, 0.5), (7, 0.7)} then

(A) $A\cup B$ = {(1, 0.3), (3, 0.4), (5, 0.2), (7, 0.8)}
(B) $A\cup B$ = {(1, 0.3), (3, 0.4), (5, 0.5), (7, 0.8)}
(C) A = {(1, 0.1), (3, 0.4), (5, 0.5), (7, 0.8)}
(D) none of these

(iv) How many ways can the letters of the word 'LEADER' be arranged?

(A) 72 (B) 144 (C) 360 (D) none of these

(v) The type of the grammar, which consists of the following productions
$s\rightarrow aA,\ A\rightarrow aAB,\ B\rightarrow b,\ A\rightarrow a$ is

(A) Type-0 (B) Type-1 (C) Type-2 (D) Type-3

(vi) Let L be a language given by $L=\{a^nb^n : n\geq 0\}$, then L^2 is equal to

(A) $\{a^nb^na^mb^m : n\geq 0, m\geq 0\}$ (B) $\{a^nb^n : n\geq 0\}$
(C) $\{a^nb^na^nb^n : n\geq 0\}$ (D) none of these

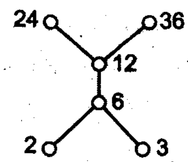
(vii) If the function $f: R \to R$ defined by $f(x) = 3x - 4$ if $x > 0$ and $f(x) = 2 - 3x$ if $x \leq 0$ then $f^{-1}(2) =$

(A) (2) (B) (0, 2) (C) (2, –2) (D) none of these

(viii) The solution of the recurrence relation $a_r - 7a_{r-1} + 10a_{r-2} = 0$ given $a_0 = 0$, $a_1 = 3$ is

(A) $5^{-r} - 2^r$ (B) $5^r + 2^r$ (C) $5^r - 2^r$ (D) none of these

(ix) Haase diagram is given below:



This is a

(A) Poset (B) Toset (C) Lattice (D) none of these

(x) Out of the following the singleton set is

(A) $A = \{x : 3x - 2 = 0, x \in Q\}$ (B) $B = \{x : x^2 - 1 = 0, x \in R\}$

(C) $C = \{x : 30x - 59 = 0, x \in N\}$ (D) $D = \{x : x^2 - 1 = 0, x \in Z\}$

where Q, R, N, Z is the set of all rational number, real number, natural number and integers respectively.

(xi) Out of the following statements the formula for tautology is

(A) $(p \vee q) \to q$ (B) $p \vee (q \to p)$ (C) $p \vee (p \to q)$ (D) $p \to (p \to q)$

(xii) A tree (acyclic connected graph) of n vertices has exactly

(A) $n - 1$ (B) n (C) $\frac{n-1}{2}$ (D) $\frac{n+1}{2}$

GROUP B
(Short Answer Type Questions)

Answer any *three* questions. $3 \times 5 = 15$

2. Draw the graphs for the following incidence matrix I and adjacency matrix A.

$$I = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}, \quad A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

3. Prove by mathematical induction, if $card(A) = n$ then $card(P(A)) = 2^n$.

4. Solve the following recurrence relation using generating function
$a_n - 6a_{n-1} + 8a_{n-2} = n-1,\ n \geq 2,\ a_0 = 1,\ a_1 = 3$

5. Draw the Hasse diagram for the divisibility relation on set A = {2, 3, 6, 12, 24, 36} also find maximal and minimal elements.

6. Show that a Binary tree with n vertices has $\frac{n+1}{2}$ pendant vertices.

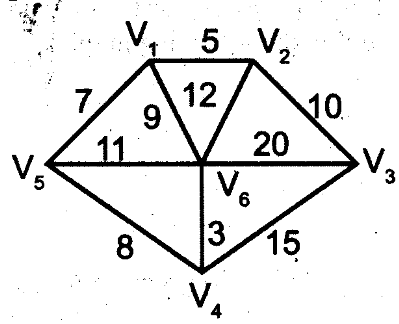
GROUP C
(Long Answer Type Questions)

Answer any *three* questions. $3 \times 15 = 45$

7. (a) Prove that a collection of sets closed under union and intersection is a lattice.

(b) Define a poset. Show that $(P(S), \subseteq)$ is a poset where $P(S)$ denotes the power set of the set $S^* = \{x, y, z\}$. Also draw the Hasse diagram for this poset.

(c) Using Prim's algorithm, find a spanning tree with minimum weight from the graph shown. Also calculate total weight of spanning tree.



8. (a) 6 boys and 6 girls are to be seated in a row. How many ways can they be seated if
   (i) all boys are to be seated together and all girls are to be seated together.
   (ii) no two girls should be seated together.

(b) Let D_{20} be the set of all positive divisors of 40. Find whether D_{20} is a Poset with respect to the relation P where aPb means a divides b. Draw the Hasse diagram of the Poset (D_{20}, P). Find the Maximal, Minimal element of D_{20}.

(c) Show by truth table that the following statement formula is a Tautology:

$$((p \rightarrow q) \wedge (q \rightarrow r) \rightarrow (p \rightarrow r)).$$

9. (a) Draw the three distinct connected graphs which are not isomorphic from the degree sequence $\{1, 3, 3, 4, 5\}$. 4+6+5

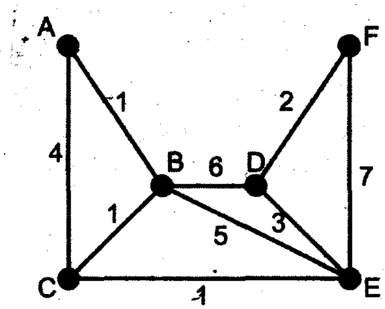
(b) Construct a Moore machine which is equivalent to the following Mealy machine

Present state	Input 0		Input 1	
	Next state	output	Next State	output
A	B	x	C	x
B	A	y	C	y
C	B	x	A	y

(c) Convert the $M = \langle \{q_0, q_1, q_2\}, \{0,1\}, \delta, q_0, q_2 \rangle$ to DFA, δ is given below:

Present State	Input 0	Input 1
	Next State	Next State
$\rightarrow q_0$	q_1	q_0, q_2
q_1	q_0	q_2
(q_2)	q_2	q_1

10.(a) By Dijkstra's algorithm find the shortest path and the length of the shortest path from the vertex A to F in the following graph : 6+5+4



(b) If $U = \{1,2,3,\ldots,10\}$, $A = \{(1,0.7), (3,0.9), (6,0.8), (7,1), (9,0.1), (10,0.6)\}$
$B = \{(1,0.2), (2, 0.6), (4,0.6), (5, 0.3), (6,0.2), (8, 0.1)\}$, find $\overline{A}$, $A \cap \overline{B}$, $A \cup B$

(c) Draw the transition diagram of an automaton M that accepts all even numbers.

11.(a) Let $f(x) = x+2$, $g(x) = x-2$ and $h(x) = 3x$ for $x \in R$, the set of real numbers. Then find $g \circ f$, $f \circ g$, $f \circ h$, $h \circ g$, $f \circ g \circ h$. 5+5+5

(b) Write short notes on any *two* of the following:
(i) Hamiltonian Graph
(ii) CNF
(iii) Planar Graph
(iv) Mealy Machine

(c) Obtain a Grammar which generates the language $L = \{a^n b^{n+1} : n \geq 0\}$