MAULANA ABUL KALAM AZAD UNIVERSITY OF TECHNOLOGY, WEST BENGAL

MM-101

DISCRETE MATHEMATICAL STRUCTURE

Time Allotted: 3 Hours | Full Marks: 70

The questions are of equal value.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.
All symbols are of usual significance.

GROUP A
(Multiple Choice Type Questions)

1. Answer any *ten* questions. $10\times1 = 10$

(i) Out of the following the singleton set is

(A) $A=\{x: 3x-2=0,\ x\in Q\}$ (B) $B=\{x: x^2-1=0,\ x\in R\}$

(C) $C=\{x: 30x-59=0,\ x\in N\}$ (D) $D=\{x: x^2-1=0,\ x\in Z\}$

where Q, R, N, Z is the set of all rational number, real number, natural number and integers respectively.

(ii) If A, B & C are any three arbitrary sets, then $A - (B \cap C)$ is

(A) $(A - B) \cup (A - C)$ (B) $(A - B) \cap (A - C)$

(C) $(A - B) \cap (C - A)$ (D) $(B - A) \cup (A - C)$

(iii) The number of arrangements of 25 objects where 7 are of the first kind, 2 are of the second kind, 3 are of the third kind and 4 are of the fourth kind is given by

(A) $\frac{25!}{7!2!3!4!}$ (B) $\frac{25!}{7!2!}$

(C) $\frac{25!}{3!4!}$ (D) none of these

(iv) Out of the following statements the formula for tautology is

(A) $(p \vee q) \rightarrow q$ (B) $p \vee (q \rightarrow p)$

(C) $p \vee (p \rightarrow q)$ (D) $p \rightarrow (p \rightarrow q)$

(v) How many ways can the letters of the word 'LEADER' be arranged?

(A) 72 (B) 144

(C) 360 (D) None of these

(vi) The mapping $f: R \rightarrow R$ defined by $f(x)=(x^2+1)^{2014}$, then the mapping is

(A) bijective (B) only injective

(C) only surjective (D) neither injective nor surjective

(vii) What is the minimum number of vertices necessary for a graph with 6 edges?

(A) 6 (B) 5

(C) 7 (D) None of these

(viii) The coefficient of X^{25} in $(X^3+X^4+X^5+...)^{25}$ is

(A) $C(9,5)$ (B) $C(5,9)$

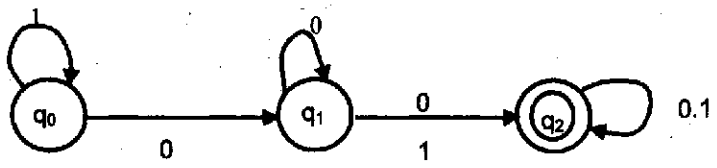
(C) $C(5,5)$ (D) $C(9,9)$

(ix) The type -3 Grammar in relation to the automata theory is known as

(A) context sensitive grammar (B) regular grammar

(C) context free grammar (D) none of these

(x) The automata



Accepts the string

(A) 11 (B) 010
(C) 111 (D) none of these

(xi) If δ is transition function of a mealy machine then for any state q and input $a, \delta(q,a) =$

(A) an input (B) a state
(C) an output (D) none of these

GROUP B
(Short Answer Type Questions)

Answer any *three* questions. $3\times5 = 15$

2. Show that $f: R \to R$ defined by $f(x) = 2x + 3$ is bijective. Find f^{-1}.

3. Prove by mathematical induction $3^{2n+1} + 2\times(-1)^n = 0 \pmod 5$.

4. Solve the recurrence relation $a_n - 2a_{n-1} = 2(n-1)$, $n \geq 1$, $a_0 = 2$ using method of undetermined coefficient.

5. A relation on the set $A = \{1, 2, 3, 4\}$ whose matrix representation is given by

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Show that whether R is an equivalence relation on A.

6. Prove that a tree with n vertices has $n-1$ edges.

GROUP C
(Long Answer Type Questions)

Answer any *three* questions. 3×15 = 45

7. (a) Prove that in a distributive lattice {L, ∨, ∧} if an element $a \in L$ has a complement, then it is unique. 5

(b) Show that the set of all positive divisors of 42 forms a poset under the relation {≤}, defined as $a \le b$, if a is a divisor of b. Draw its Hasse diagram. 5

(c) Solve the recurrence relation $a_n - 9a_{n-1} + 20a_{n-2} = 0, n \ge 2, a_0 = -3, a_1 = -1.$ 5

8. (a) Obtain the CNF of ¬(p →(q ∧ r)). 4+6+5

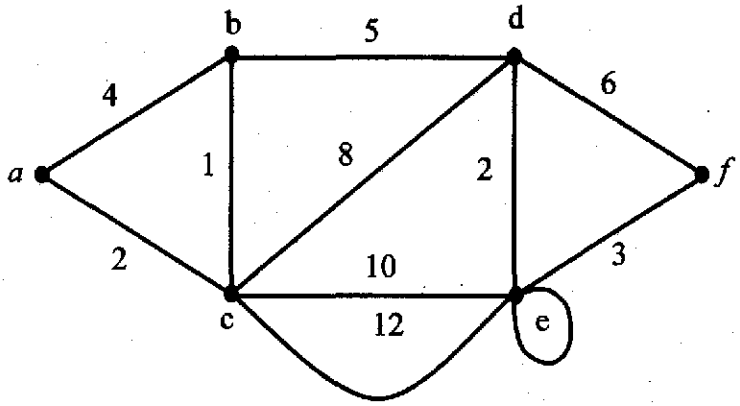
(b) Construct a Moore machine which is equivalent to the following Mealy machine

Present state	Input 0		Input 1	
	Next state	output	Next state	output
A	B	X	C	Y
B	A	Y	C	X
C	B	X	A	Y

(c) Convert the $M = \langle \{q_0, q_1, q_2\}, \{0,1\}, \delta, q_0, q_2 \rangle$ to DFA, δ is given below:

Present State	Input 0	Input 1
	Next State	Next State
→ q_0	q_1	q_2
q_1	q_0, q_1	q_2
(q2)	q_2	q_1

9. (a) By Dijkstra's algorithm find the shortest path and the length of the shortest path from the vertex a to f in the following graph: 7+4+4

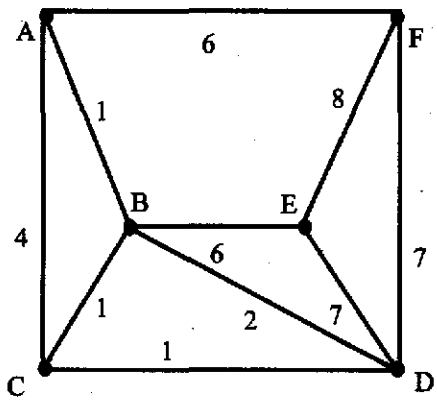


(b) If $U = \{2, 3, 4, \ldots, 9\}$ and the two fuzzy sets are
$A = \{(3, 0.9), (6, 0.8), (7,1), (9, 0.1)\}$.
$B = \{(2, 0.6), (4, 0.6), (5, 0.3), (6, 0.2), (8, 0.1)\}$. Find $\overline{B}, \overline{A} \cap \overline{B}, A \cup B$.

(c) Draw the transition diagram of automation M that accepts all even numbers.

10.(a) Prove that the maximum number of edges of a graph with n vertices and k components is $\frac{(n-k)(n-k+1)}{2}$. 7

(b) Find the minimal spanning tree of the following graph by Kruskal's algorithm (show intermediate steps). 8



11.(a) Find the coefficient of $15\,x^{15}$ in $(x + x^2 + x^3)^2\,(x^2 + x^3 + x^4 + x^5 + \ldots\ldots\infty)^3$. 5

(b) Write an equivalent formula for $\sim(p \leftrightarrow (q \rightarrow (r \vee p)))$ which does not contain any conditional ($\rightarrow$) and bi-conditional ($\leftrightarrow$) symbols. 5

(c) Test whether the following two graphs are isomorphic to each other or not? 5