ENGINEERING & MANAGEMENT EXAMINATIONS, DECEMBER – 2006

DISCRETE MATHEMATICAL STRUCTURE

SEMESTER – 1

Time : 3 Hours] [Full Marks : 70

Graph sheet is provided on Page 31.

Group – A

(Multiple Choice Questions)

1. Choose the correct alternatives for any *ten* of the following questions : $10 \times 1 = 10$

i) Out of the following the singleton set is

a) $A = \{ x : 3x - 2 = 0, x \in Q \}$

b) $B = \{ x : x^2 - 1 = 0, x \in R \}$

c) $C = \{ x : 30x - 59 = 0, x \in N \}$

d) $D = \{ x : x^2 - 1 = 0, x \in Z \}$

where, Q is the set of all rational numbers, R is the set of all real numbers, N is set of all natural numbers and Z is the set of all integers.

ii) If A, B & C are any three arbitrary sets, then $A - (B \cap C)$ is

a) $(A - B) \cup (A - C)$

b) $(A - B) \cap (A - C)$

c) $(A - B) \cap (C - A)$

d) $(B - A) \cup (A - C)$.

iii) The mapping $f : R \to R$ defined by $f(x) = (x^2 + 1)^{2006}$. Then the mapping is

a) bijective

b) only injective

c) only surjective

d) neither injective nor surjective.

iv) Out of the follwoing statements the formula for tautology is

a) $(p \vee q) \rightarrow q$

b) $p \vee (q \rightarrow p)$

c) $p \vee (p \rightarrow q)$

d) $p \rightarrow (p \rightarrow q)$.

v) The total number of different ways that 3 letters can be posted into six letter boxes is

a) 6^3

b) 3^6

c) 18

d) 27.

vi) If N = set of all natural numbers, then $f: N \rightarrow N$ defined as

$$f(n) = \begin{cases} 2n, & \text{if } n \text{ is even} \\ n, & \text{if } n \text{ is odd} \end{cases} \quad \text{is}$$

a) onto

b) one-one

c) both of (a) & (b)

d) none of these.

vii) The type of the grammar, which consists of the following productions

$s \rightarrow aA,\ A \rightarrow aAB,\ B \rightarrow b,\ A \rightarrow a$ is

a) type – 0

b) type – 1

c) type – 2

d) type – 3.

viii) A tree (acyclic connected graph) of n vertices has exactly

a) $n - 1$

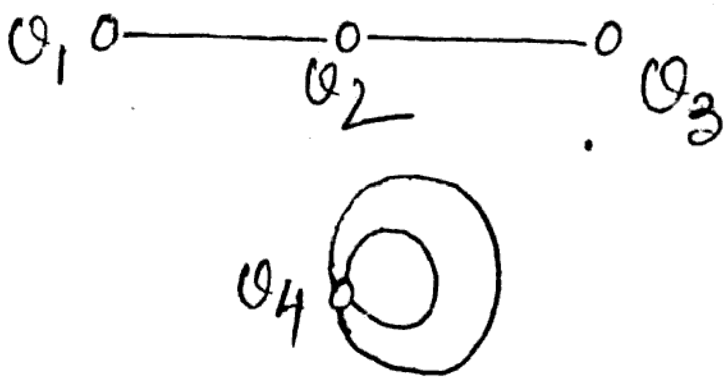
b) n

c) $\frac{n-1}{2}$

d) $\frac{n+1}{2}$

edges.

ix) In the following graph :



deg (v_4) [degree of the vertex v_4] is

a) 2 b) 0

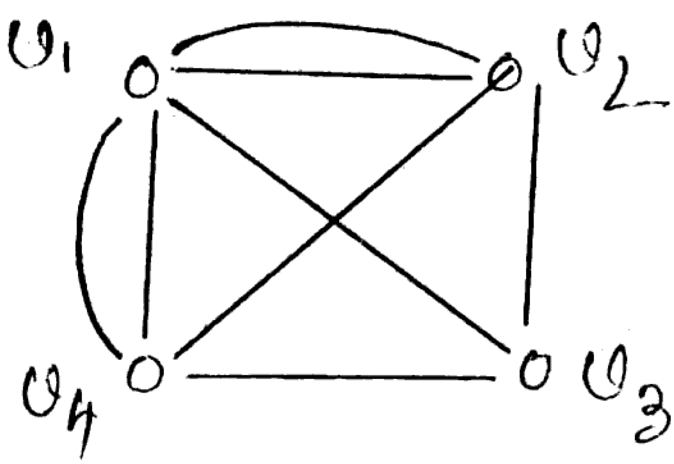
c) 4 d) 5.

x) Number of relations from A = { a, b, c } to B = { 1, 2 } are

a) 6 b) 9

c) 5 d) 8.

xi) The adjacency matrix of the following graph is



a) $\begin{bmatrix} 0 & 2 & 1 & 2 \\ 2 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 2 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

d) none of these.

xii) The pre-image of the point $y = 0$, with respect to the mapping $f : R \to R$, defined by, $y = f(x) = \sin x$, $\forall x \in$ R, is

a) $n\pi$, $n = 0, 1, 2, 3, \ldots\ldots$

b) $n\pi$, $n = 0, -1, -2, -3, \ldots\ldots$

c) $n\pi$, $n = 0, \pm 1, \pm 2, \ldots\ldots$

d) $\frac{n\pi}{2}$, $n = 0, \pm 1, \pm 2, \ldots\ldots$.

xiii) The generating function of the following sequence

$\{ 0, 0, 1, 1, 1, 1, 1, \ldots\ldots \}$

is

a) $x^2 (1 + x)^{-1}$

b) $x^2 (1 + x)^{-2}$

c) $x (1 - x)^{-1}$

d) $x^2 (1 - x)^{-1}$.

xiv) If a tree has 10 vertices, then number of its edges is

a) 8

b) 11

c) 10

d) none of these.

Group - B

(Short Answer Questions)

Answer any *three* of the following questions. $3 \times 5 = 15$

2. i) By using "Principle of Mathematical Induction", prove that $n^3 + 2n$ is divisible by 3, for $n \geq 1$. 2

   ii) If n = set of all natural numbers and $f: N \to N$ is given by $f(n) = n - (-1)^n$ for $n \in N$, examine if f is bijective. 3

3. i) Define Convex Fuzzy set with an example. 2

   ii) Consider following two Fuzzy sets :

   $\mu_1 : \{ (4, 0, 2), (6, 0, 4), (8, 0, 6), (10, 1) \}$

   $\mu_2 : \{ (1, 0, 9), (2, 0, 7), (3, 0, 5) \}$

   then determine μ_2' & $\mu_1 \cup \mu_2'$. 3

4. Let $A = \{ x \in R : x \neq 2 \}$ & $B = \{ x \in R : x \neq 1 \}$, and let the following two functions

   $f: A \to B$, & $g: B \to A$, are defined by

   $f(x) = \dfrac{x}{x-2}$, $\forall x \in A$ and $g(x) = \dfrac{2x}{x-1}$, $\forall x \in B$

   then find the following :

   i) $f \circ g$ 2

   ii) Are the two functions f and g invertible ? 3

5. A light bulb is located at a staircase in a two-storied building and there are two switches, one in the ground floor and the other in the first floor. Design a switching circuit connecting the switches and the bulb in such a way that either switch may be used to control the light independently of the state of the other. 5

6. i) State the Generalized Pigeonhole Principle.

   ii) Suppose a laundry bag contains many red, white and blue socks. Find the minimum number of socks that one needs to choose in order to get two socks of the same colour. 2 + 3

7. Solve the following difference equation with the help of generating function :

$a_n - a_{n-1} = 3(n-1)$, $n \geq 1$ and where $a_0 = 2$. 5

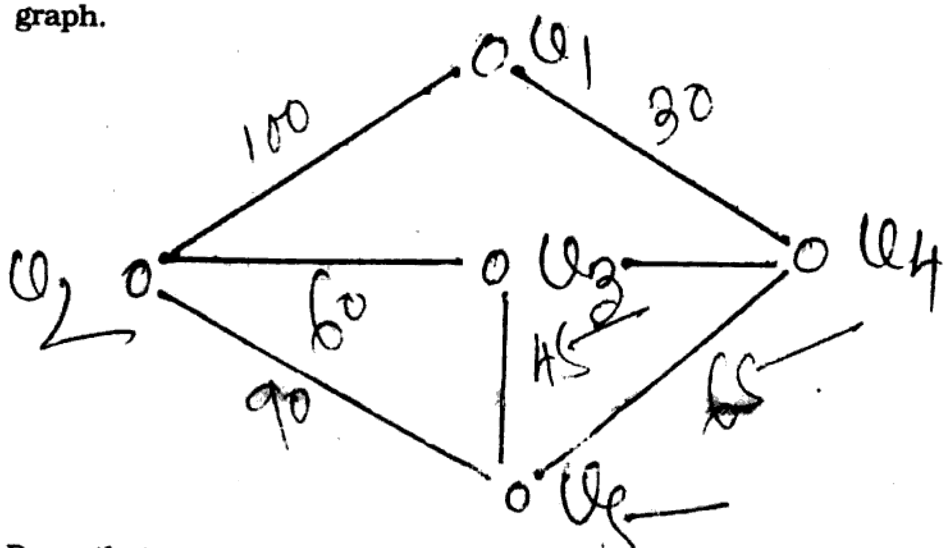
8. Show that a connected graph of n vertices and $(n-1)$ edges is a tree. 5

Group - C

(Long Answer Questions)

Answer any *three* of the following questions. $3 \times 15 = 45$

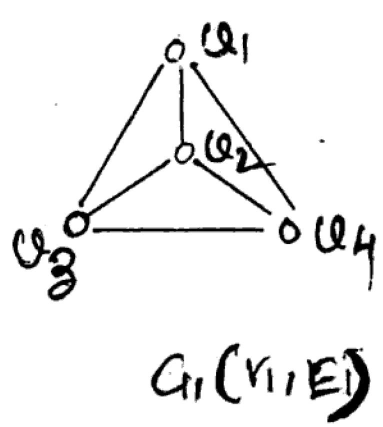
9. i) Define minimum Spanning Tree of a graph with an example. Apply Kruskal Algorithm to determine the minimum spanning tree of the following weighted graph.

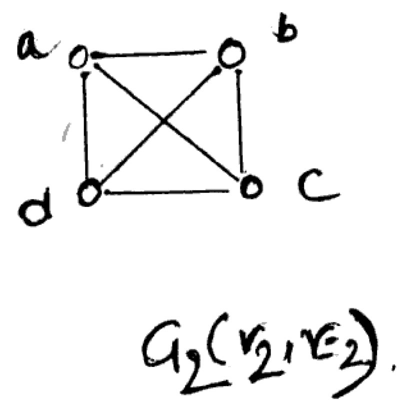


ii) Prove that a simple graph $G(V, E)$ has a spanning tree iff $G(V, E)$ is connected graph.

10 + 5

10. i) Explain the term 'graph isomorphism' by an example. Test whether the following two graphs $G_1(V_1, E_1)$ & $G_2(V_2, E_2)$ are isomorphic to each other or not ?





ii) Solve the following recurrence relation :

$a_{n+2} = 6a_{n+1} - 9a_n + 3*2^n + 7*3^n, n \geq 0$

with $a_0 = 1, a_1 = 4$.

iii) Prove that in a Distributive Boolean Algebra ($B, +, ..., /$) if

$x_1 \vee x_2 = x_1 \vee x_3$ &

$x_1 \wedge x_2 = x_1 \wedge x_2$

then $x_2 = x_3$, where $x_1, x_2, x_3, x_4 \in B$. 5 + 5 + 5

11. i) Define the following by examples :

a) D.F.A.

b) N.D.F.A.

ii) Determine a D.F.A. from the N.D.F.A. $M = (\{q_0, q_1\}, \{0, 1\}, \delta, q_0, \{q_1\})$, with the state transition function δ as given in the table :

States	Input	
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
q_1 (final states)	Φ	$\{q_0, q_1\}$

5 + 10

12. i) Define Moore machine and Mealy machine. Construct a Mealy machine which is equivalent to the Moore machine given in the following table :

Present States	Next States		Output
	0	1	
$\rightarrow q_0$	q_1	q_2	1
q_1	q_3	q_2	0
q_2	q_2	q_1	1
q_3	q_0	q_3	1

ii) Find the number of parallelograms formed by intersecting of two sets of m and n parallel straight lines in a plane. 10 + 5

13. i) If all the vertices of an undirected graph are each of odd degree k, show that the number of edges of the graph is a multiple of k.

ii) Let $A = \{ 1, 2, 3 \}$, $B = \{ w, x, y, z \}$ and $f : A \rightarrow B$.

a) How many functions f are there ?

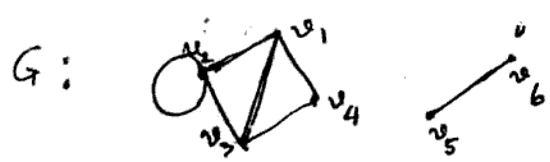
b) How many of them are injective ? 7 + 8

14. i) Define Adjacency matrix of a Graph. A graph G has following as Adjacency matrix :

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}.$$

Draw the Graph and examine if it is connected.

ii) Find Adjacency matrix of Graph G as under.



8 + 7