ENGINEERING & MANAGEMENT EXAMINATIONS, DECEMBER – 2007

DISCRETE MATHEMATICAL STRUCTURE

SEMESTER – 1

Time : 3 Hours] [Full Marks : 70

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) Null set is the subset of

a) universal set b) universe of discourse

c) every set d) none of these.

ii) Cardinality of the power set of a non-empty set A is

a) $2^{|A|}$ b) $2 \mid A \mid$

c) $\mid A \mid^2$ d) none of these.

iii) A partial order relation is

a) always antisymmetric b) sometimes antisymmetric

c) irreflexive d) none of these.

iv) Degrees contributed by a oop to a vertex in a graph G is

a) 2 b) 1

c) 0 d) none of these.

v) A simple graph

a) does not possess a loop b) must possess a loop

c) is necessarily a multigraph d) is necessarily a pseudograph.

vi) The relation $\subset$ is

a) irreflexive b) antisymmetric

c) asymmetric d) all of these.

vii) If $f : X \to Y$ and $g : Y \to Z$ are injective, then $g \circ f$ is

a) injective b) surjective

c) invertible d) none of these.

viii) $\{a, b\} \leq V_T$ and $S \in V_N$, then $S \to ab$ is a

a) type-0 grammar b) type-1 grammar

c) type-2 grammar d) type-3 grammar.

ix) Minimum height of an n-vertex binary tree is

a) $\frac{n-1}{2}$ b) $\frac{n+1}{2}$

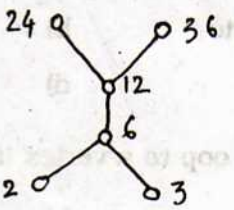
c) $\lceil \log_2 (n+1) - 1 \rceil$ d) $\lfloor \log_2 (n+1) - 1 \rfloor$.

x) Let A be a set with 10 distinct elements. How many different binary relations on A are there ?

a) 100 b) 100^2

c) 2^{100} d) none of these.

xi) Hasse diagram is given below :



This is a

a) Poset b) Toset

c) Lattice d) none of these.

xii) Six boys and four girls can sit in a row in

a) $6! \times 4!$ ways b) $2 \times 6! \times 4!$ ways

c) 2^{24} ways d) none of these.

xiii) Let $|A| = |B| = n$, where $|A|$ denotes number of elements of A. Then the number of bijective mappings from A to B is

a) 1 b) n^n

c) $n!$ d) can't be said.

xiv) Let $f: Z \to Z$ given by $f(x) = 2x$, $x \in Z$. Then

a) f is injective but not surjective b) f is surjective but not injective

c) f is bijective d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Define distributive lattice. Prove that in a distributive lattice

$$(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) = (a \vee b) \wedge (b \vee c) \wedge (c \vee a).$$

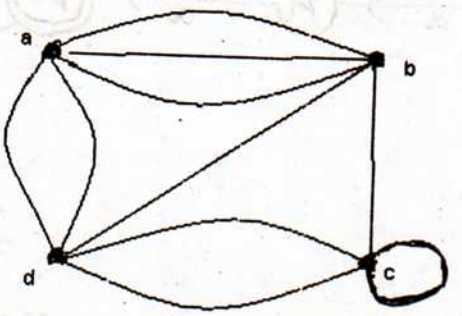
3. Prove that number of vertices of odd degree in a graph $G = (V, E)$ is always even.

4. Find the particular solution of the difference equation :

$$a_{r+2} - 5a_{r+1} + 6a_r = 5^r$$

5. Prove that the number of circular permutations of n different objects is $(n-1)!$

6. Define adjacency matrix of a simple graph $G = (V, E)$. Write down the adjacency matrix for the following undirected graph :



7. Use mathematical induction to prove that $(3^n + 7^n - 2)$ is divisible by 8, here $n \geq 1$.

GROUP – C

(Long Answer Type Questions)

Answer any *three* questions. $3 \times 15 = 45$

8. a) Determine the union of the following two fuzzy sets :

$A = \left\{ \frac{4}{0{\cdot}1}, \frac{6}{0{\cdot}5}, \frac{8}{0{\cdot}6}, \frac{10}{0{\cdot}7} \right\}$ and $B = \left\{ \frac{0}{0{\cdot}4}, \frac{2}{0{\cdot}6}, \frac{4}{1}, \frac{6}{1}, \frac{8}{0{\cdot}6}, \frac{10}{0{\cdot}5} \right\}$.

b) Show that the mapping $f : R - \{\sqrt{2}\} \to R$ defined by $f(x) = \frac{x}{x^2 - 2}$, $x \neq \sqrt{2}$ is surjective but not injective.

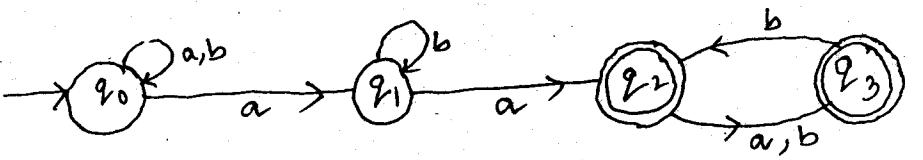
c) A simple graph with n vertices and k components can have at most $(n-k)(n-k+1)/2$ edges. 3 + 6 + 6

9. a) Design an FA which accepts the language

$L = \{ w / w$ has both an even number of 0's and even number of 1's over alphabet $\Sigma = \{0, 1\} \}$.

b) Design an *FA* which accepts set of strings containing exactly four 1's in every string over alphabet $\Sigma = \{0, 1\}$.

c) Convert the following NFA into DFA.



5 + 4 +

10. a) Given the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ and the initial conditions $a_0 = 2$, $a_1 = 5$ and $a_2 = 15$, find the solution.

b) Find solution for the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2}$ with initial conditions $a_0 = 1$ and $a_1 = 6$. 8 +

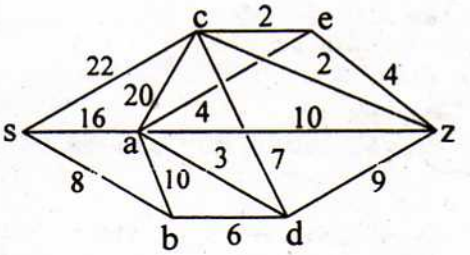
11. a) If $P(S)$ is the power set of a set S and $\cup$ and $\cap$ are taken as the join and meet, prove that $\{P(S), \subseteq\}$ is a lattice.

b) Define partially ordered set. Can you say all partially ordered sets are lattice ? Justify.

c) Give an example of a relation ρ on A (described by you) which is symmetric and transitive but not reflexive with justification. 5 + 6 + 4

12. a) State Dijkstra's algorithm for shortest path problem.

b) Use Dijkstra's algorithm to find the shortest path between the vertices from s to z in the following graph.



5 + 10

13. Write short notes on any *three* of the following : 3 × 5

a) Bipartite graph

b) DFA

c) NFA

d) Moore machine

e) Mealy machine.

END