## CS/MCA/SEM-1/M (MCA)-101/2011-12

## 2011

## DISCRETE MATHEMATICAL STRUCTURE

Time Allotted: 3 Hours

Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words

as far as practicable.

# GROUP - A ( Multiple Choice Type Questions )

1. Choose the correct alternatives for any ten of the following:

 $10 \times 1 = 10$ 

- i) The number of arrangements of 25 objects where 7 are of the first kind, 12 are of the second kind, 3 are of the third kind and 4 are of the fourth kind is given by
  - a) (25!)/(7!2!3!4!)
- b) (25!)/(7!2!)

c) (25!)/(3!4!)

- d) none of these.
- ii) Which one is a singleton set?
  - a)  $\{0, 1\}$

b) {1, 11, 111}

http://www.makaut.com/

d) none of these.

| iii)                                     | Wł   | nat is the minimum no            | o. of      | vertices | necessary    | for a |
|--|--|----------------------------------|------------|----------|--------------|-------|
|  | graph with 6 edges ?                                 |                                  |            |          |              |       |
|  | a)   | 6                                | b)         | 5        | *            |       |
|  | c)   | 7                                | d)         | none o   | of these.    |       |
| iv)                                      | In a binary tree, the parent may have                |                                  |            |          |              |       |
| a) right child                           |  |                                  |            | 1        |              |       |
|  | b)   | left child                       |            |          |              |       |
|  | c) both right and left children                      |                                  |            |          |              |       |
| d) right or left or both children.       |  |                                  |            |          |              |       |
| v) Null set is the subset of             |  |                                  |            |          |              |       |
|  | a)   | universal set                    | b)         | univers  | se of discou | ırse  |
|  | c)   | every set                        | d)         | none of  | f these.     |       |
| vi)                                      | Cardinality of the power set of a non-empty set A is |                                  |            |          |              |       |
|  | a)   | 2 A                              | b)         | 2 A      |              |       |
|  | c)   | A  <sup>2</sup>                  | d)         | none of  | these.       |       |
| vii) Which of the following is not true? |  |                                  |            |          |              |       |
|  | a)   | $A \oplus B = (A - B) \cap (B -$ | <i>A</i> ) |          |              |       |
|  | b)   | $A \cup \widetilde{A} = U$       |            |          |              |       |

d)  $A-B=\overline{B}-\overline{A}$ .

c)  $\overline{A} \cup \overline{B} = \overline{(A \cap B)}$ 

| viii)  | Α | spanning | tree | has   |
|--------|---|----------|------|-------|
| * **** |   | Sharrang | ~~~  | ***** |

- a) only one circuit
- b) two circuits

c) no circuit

d) none of these.

ix) The type-3 Grammar in relation to the automata theory is known as

- a) context sensitive grammar
- b) regular grammar
- c) context free grammar
- d) none of these.

x) If p: 'Anil is rich' & q: 'Kanchan is poor' then the symbolic form for the statement Either Anil or Kanchan is rich' is

a)  $p \vee q$ 

b)  $p \lor \sim q$ 

c)  $\sim p \vee q$ 

d)  $\sim (p \wedge q)$ .

xi) How many arrangements are possible by the word "LETTER"?

a) 720

b) 360

c) 60

d) 180.

A pendant vertex has degree xii)

> 1 a)

b) 2

c) 3

none of these. d)

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#### **GROUP -- B**

## ( Short Answer Type Questions )

Answer any three of the following.  $3 \times 5 = 15$ 

- Let f(x) = x + 2, g(x) = x 2 and h(x) = 3x for  $x \in R$ , the set 2. of real numbers. Then find  $g \circ f$ ,  $f \circ f$ ,  $f \circ h$ ,  $h \circ g$ , fogoh.
- Define distributive lattice. Prove that in a distributive lattice 3.  $(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) = (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$
- Let G be a graph with n vertices and e edges. Prove that G 4. has a vertex of degree m such that  $m \ge \frac{2e}{a}$ .
- By mathematical induction prove that 5.  $3^{2n+1} + (-1)^n 2 = 0 \pmod{5}$ .
- Define a planar graph. Show that  $K_5$  is non-planar. 6.

#### **GROUP - C**

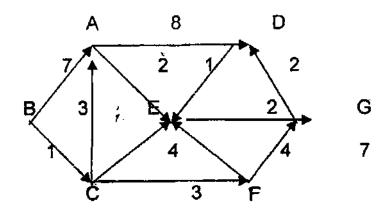
### (Long Answer Type Questions)

Answer any three of the following.  $3 \times 15 = 45$ 

- 7. a) Obtain equivalent disjunctive normal form of  $\sim G \land (H \leftrightarrow G)$ .
  - b) Solve the following recurrence relation using generating function:

$$a_n - 2a_{n-1} + a_{n-2} = 2^{n-2}$$
 for  $n \ge 2$  and  $a_0 = 1$ ,  $a_1 = 5$ .

- c) Determine whether the posets ({ 1, 2, 3, 4, 5 },|) and ({ 1, 2, 4, 8, 16 },|) are lattices. Here the relation '|' implies "divides".
- 8. a)



Find the shortest distance between B and G applying Dijkstra's Algorithm.

- b) Give an example of a relation on A (described by you) which is symmetric and transitive but not reflexive (with justification).
- c) Show that  $(p \lor q)^{\land} (\sim p^{\land} \sim q)$  is a contradiction.

- 9. a) Prove that if there is one and only path between every pair of vertices in a graph G, then G is a tree.
  - b) Construct the truth table for:

$$(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r)).$$

- c) If P (S) is the power set of a set S and  $\cup$  and  $\cap$  are taken as the join and meet, prove that  $(P(S), \subseteq)$  is a lattice.
- 10. a) Use mathematical induction to prove that  $n^3 + 2n$  is divisible by 3.
  - b) What do you mean by disjunction and conjunction?
  - c) Convert the given Moore Machine to its equivalent Mealy

    Machine:

| Present               | Next state |           | Output |
|-----------------------|------------|-----------|--------|
| state                 | Input a=0  | Input a=1 |        |
| $q_0$                 | $q_3$      | $q_1$     | 0      |
| <b>q</b> <sub>1</sub> | $q_1$      | $q_2$     | 1      |
| $q_2$                 | $q_2$      | $q_3$     | 0      |
| $q_3$                 | $q_3$      | $q_0$     | 0      |

# CS/MCA/SEM-1/M (MCA)-101/2011-1

# 11. a) Prove the following equivalence:

$$p \leftrightarrow (p \land q) \lor (p \land \neg q)$$

b) Construct a DFA from the NFA:

| State | Input (0) | Input (1) | Output |
|-------|-----------|-----------|--------|
| Α     | В         | В,С       | 0      |
| В     | A, C      |           | 0      |
| С     | Α         | B, C      | 1      |

c) Write a short note on Fuzzy sets.