Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/MCA/SEM-1/M (MCA)-101/2011-12

2011
DISCRETE MATHEMATICAL STRUCTURE

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i) The number of arrangements of 25 objects where 7 are of the first kind, 12 are of the second kind, 3 are of the third kind and 4 are of the fourth kind is given by

a) (25!)/(7!2!3!4!) b) (25!)/(7!2!)

c) (25!)/(3!4!) d) none of these.

ii) Which one is a singleton set ?

a) {0, 1} b) {1, 11, 111}

c) {0} d) none of these.

iii) What is the minimum no. of vertices necessary for a graph with 6 edges ?

a) 6 b) 5

c) 7 d) none of these.

iv) In a binary tree, the parent may have

a) right child

b) left child

c) both right and left children

d) right or left or both children.

v) Null set is the subset of

a) universal set b) universe of discourse

c) every set d) none of these.

vi) Cardinality of the power set of a non-empty set A is

a) $2^{|A|}$ b) $2|A|$

c) $|A|^2$ d) none of these.

vii) Which of the following is not true ?

a) $A \oplus B = (A - B) \cap (B - A)$

b) $A \cup \overline{A} = U$

c) $\overline{A} \cup \overline{B} = \overline{(A \cap B)}$

d) $A - B = \overline{B} - \overline{A}$.

viii) A spanning tree has

a) only one circuit b) two circuits

c) no circuit d) none of these.

ix) The type–3 Grammar in relation to the automata theory is known as

a) context sensitive grammar

b) regular grammar

c) context free grammar

d) none of these.

x) If p : 'Anil is rich' & q : 'Kanchan is poor' then the symbolic form for the statement 'Either Anil or Kanchan is rich' is

a) $p \vee q$ b) $p \vee \sim q$

c) $\sim p \vee q$ d) $\sim (p \wedge q)$.

xi) How many arrangements are possible by the word "LETTER" ?

a) 720 b) 360

c) 60 d) 180.

xii) A pendant vertex has degree

a) 1 b) 2

c) 3 d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Let $f(x) = x + 2$, $g(x) = x - 2$ and $h(x) = 3x$ for $x \in R$, the set of real numbers. Then find g o f, f o f, f o h, h o g, f o g o h.

3. Define distributive lattice. Prove that in a distributive lattice

$$(a \wedge b) \vee (b \wedge c) \vee (c \wedge a) = (a \vee b) \wedge (b \vee c) \wedge (c \vee a)$$

4. Let G be a graph with n vertices and e edges. Prove that G has a vertex of degree m such that $m \geq \frac{2e}{n}$.

5. By mathematical induction prove that

$3^{2n+1} + (-1)^n 2 = 0 \pmod 5$.

6. Define a planar graph. Show that K_5 is non-planar.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

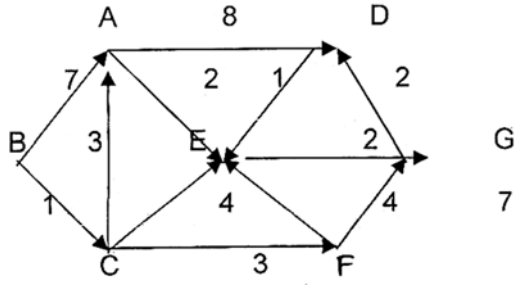
7. a) Obtain equivalent disjunctive normal form of $\sim G \wedge (H \leftrightarrow G)$.

   b) Solve the following recurrence relation using generating function :

   $a_n - 2a_{n-1} + a_{n-2} = 2^{n-2}$ for $n \geq 2$ and $a_0 = 1$, $a_1 = 5$.

   c) Determine whether the posets $(\{ 1, 2, 3, 4, 5 \}, |)$ and $(\{ 1, 2, 4, 8, 16 \}, |)$ are lattices. Here the relation ' | ' implies "divides".

8. a)



   Find the shortest distance between B and G applying Dijkstra's Algorithm.

   b) Give an example of a relation on A (described by you) which is symmetric and transitive but not reflexive (with justification).

   c) Show that $(p \vee q) \wedge (\sim p \wedge \sim q)$ is a contradiction.

9. a) Prove that if there is one and only path between every pair of vertices in a graph G, then G is a tree.

b) Construct the truth table for :

$(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$.

c) If P (S) is the power set of a set S and $\cup$ and $\cap$ are taken as the join and meet, prove that (P(S), $\subseteq$) is a lattice.

10. a) Use mathematical induction to prove that $n^3 + 2n$ is divisible by 3.

b) What do you mean by disjunction and conjunction ?

c) Convert the given Moore Machine to its equivalent Mealy Machine :

Present state	Next state		Output
	Input a=0	Input a=1	
q_0	q_3	q_1	0
q_1	q_1	q_2	1
q_2	q_2	q_3	0
q_3	q_3	q_0	0

11. a) Prove the following equivalence :

$$p \leftrightarrow (p \wedge q) \vee (p \wedge \neg q)$$

b) Construct a DFA from the NFA :

State	Input (0)	Input (1)	Output
A	B	B,C	0
B	A, C	----	0
C	A	B, C	1

c) Write a short note on Fuzzy sets.