Name : ....

Roll No. : ....

Invigilator's Signature : ....

CS/MCA/SEM-1/M(MCA)-101/2009-10

2009

DISCRETE MATHEMATICAL STRUCTURES

Time Allotted : 3 Hours Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10

i) The number of arrangements of 25 objects where 7 are of the first kind, 12 are of the second kind, 3 are of the third kind and 4 are of the fourth kind is given by

a) $\frac{25!}{7!2!3!4!}$ b) $\frac{25!}{7!2!}$

c) $\frac{25!}{3!4!}$ d) none of these.

ii) The coefficient of X^{25} in $\left(X^3 + X^4 + X^5 + \ldots\right)^5$ is

a) $C(9, 5)$ b) $C(5, 9)$

c) $C(5, 5)$ d) $C(9, 9)$.

iii) Which one is a singleton

a) { 0, 1 } b) { 1, 11, 111 }

c) { 0 } d) None of these.

iv) If A is a proper subset of a non-empty set S and two subsets A and A' are non-empty, then which one is true ?

a) $A \cup A' = S$ b) $A \cap A' = \phi$

c) both (a) & (b) d) None of these.

v) In the following graph

deg (V_3) is

a) 1 b) 0

c) 2 d) 5.

vi) If A and B are two subsets, then A and B are said to be disjoint if

a) $A \cap B = \phi$ b) $A \cup B = \phi$

c) $A - B = \phi$ d) none of these.

vii) If a set $S = \{ 1, 2, 3 \}$, then the power set of S is

a) $\{ \phi, S \}$ b) $\{ \phi \}$

c) $\{ S \}$ d) none of these.

viii) How many ways can the letters of the word 'LEADER" be arranged ?

a) 72 b) 144

c) 360 d) None of these.

ix) In a binary tree, the parent may have

a) right child

b) left child

c) both right and left childs

d) right or left or both childs.

x) The Fuzzy logic is based on mapping the universe of discourse to

a) [0, 1] b) (0, 1)

c) { 0, 1 } d) none of these.

xi) In Prime's Algorithm, the weight of non-existing edge is taken as

a) 0 b) + ∞

c) 1 d) none of these.

xii) Let L be a language given by $L = \{a^n b^n : n \geq 0\}$, then L^2 is equal to

a) $\{a^n b^n a^m b^m : n \geq 0, m \geq 0\}$

b) $\{a^n b^n : n \geq 0\}$

c) $\{a^n b^n a^m b^m : n \geq 0\}$

d) none of these.

xiii) If n be the number of vertices, e be the number of edges and k be the number of components of a graph G, then

a) $e \geq n + k$
b) $e \geq n - k$
c) $e \leq n - k$
d) none of these.

GROUP – B
(Short Answer Type Questions)

Answer any *three* of the following. 3 × 5 = 15

2. Consider the language $L = \{0^n 1^n : n \neq m\}$, find a context free grammar G which generates L.

3. Show that the maximum number of edges in a simple graph with n vertices is $n (n - 1) / 2$.

4. Let A be some fixed 10-element subset of $S = \{1, 2, 3, 4, 5, ... 50\}$. Show that A possesses two different 5-element subsets, the sums of whose elements are equal.

5. Solve the following using generating function :

   $a_n - a_{n-1} = 3(n-1), n \geq 1$, and where $a_0 = 2$.

6. Find the coefficient of x^{18} in

   $\left(x + x^2 + x^3 + x^4 + x^5\right)\left(x^2 + x^3 + x^4 + x^5 + ...\right)^5$.

7. Obtain equivalent disjunctive normal form of $\sim G \wedge (H \Leftrightarrow G)$.

8. Design a finite state machine that performs serial addition.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

9. a) Let $X = \{ 1, 2, 3, \ldots , 7 \}$ and

   $R = \{ (x, y) : x - y$ is divisible by $3 \}$. Prove that R is an equivalence relation and draw the relation graph.

   b) Find the transitive closure of a relation R on the set $\{ a, b, c \}$, whose relation matrix M_R is given as

   $$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}.$$

   7 + 8

10. a) Prove that 21 divides $4^{n+1} + 5^{2n-1}, \forall n > 0$.

    b) Let M be the finite state machine with state table appearing in the following table :

	f			g		
S \ A	a	b	c	a	b	c
S_0	S_0	S_0	S_0	0	1	0
S_1	S_0	S_0	S_0	1	1	1
S_2	S_0	S_0	S_0	1	0	0

    i) Find the input set A, the state set S, the output set O, and initial state of M.

    ii) Draw the state diagram of M.

    Find the output string for the input string *aabbcc*.

    5 + 10

11. a) Prove that if there is one and only path between every pair of vertices in a graph G, then G is a tree.

b) Describe Kruskal's algorithm to find the Minimal spanning tree in a graph G. Use this algorithm to find minimal spanning tree for the following graph :

c) Prove that a simple graph with n vertices and k components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges. 5 + 5 + 5

12. a) Prove that a simple graph has a spanning tree iff it is connected.

b) Find the sequence $\{y_x\}$ having the generating function G, given by $G(x) = \frac{3}{1-x} + \frac{1}{1-2x}$.

c) By mathematical induction prove that $3^{2n+1} + (-1)^n 2 \equiv 0 \pmod 5$. 5 + 5 + 5

13. a) Let $A = \{ a, b, c \}$, find L^* and L^+ where

i) $L = \{b^2\}$

ii) $L = \{a, b\}$

b) Prove the following identities :

i) $\lambda + 1^*(011)^*\left(1^*(011)\right)^* = (1 + 011)^*$

ii) $\left(1+00^*1\right)+\left(1+00^*1\right)\left(0+10^*1\right)^*\left(0+10^*1\right)=0^*1\left(0+10^*1\right)^*$

c) Draw the transition diagram of the non-deterministic finite-state automaton whose next state is given below :

S \ A	0	1
S_0	$\{S_0, S_1\}$	$\{S_2\}$
S_1	Φ	$\{S_1\}$
S_2	$\{S_1, S_2\}$	Φ

5 + 5 + 5

14. a) Show that $(p \vee q) \wedge (- p \wedge \sim q)$ is a contradiction.

b) Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \Rightarrow R$, $P \Rightarrow M$ and $\sim M$.

c) Determine a DFA from the NDFA $M = (\{q_0, q_1\}, \{0, 1\}, \delta, q_0, \{q_1\})$, with the state transition function δ as given in the following table :

States	Input	
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
q_1 (Final state)	Φ	$\{q_0, q_1\}$

5 + 5 + 5

15. a) Prove that a simple graph G (V, E) has a spanning tree iff G (V, E) is connected graph.

b) Define the following by example :

i) DFA

ii) NDFA

c) If (A, $\leq$) and (B, $\leq$) are posets, then prove that $\{(A \, x \, B, \leq)\}$ is a poset with partial order $\leq$ defined as $(a, b) \leq (a, b)$, if $a \leq a$ in A and $b \leq b$ in B. 5 + 5 + 5