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### DISCRETE MATHEMATICAL STRUCTURES

Time Allotted: 3 Hours Full Marks: 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

# GROUP - A ( Multiple Choice Type Questions )

- 1. Choose the correct alternatives for any ten of the following:  $10 \times 1 = 10$ 
  - i) The number of arrangements of 25 objects where 7 are of the first kind, 12 are of the second kind, 3 are of the third kind and 4 are of the fourth kind is given by
    - a)  $\frac{25!}{7!2!3!4!}$
- b)  $\frac{25!}{7!2}$

c)  $\frac{25!}{3!4!}$ 

- d) none of these.
- ii) The coefficient of  $X^{25}$  in  $(X^3 + X^4 + X^5 + ...)^5$  is
  - a) C(9,5)
- b) C(5, 9)
- c) C(5,5)
- d) C(9, 9).

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- iii) Which one is a singleton
  - a)  $\{0, 1\}$

b) {1, 11, 111

c) { 0 }

- d) None of these.
- iv) If A is a proper subset of a non-empty set S and two subsets A and A' are non-empty, then which one is true?
  - a)  $A \cup A' = S$
- b)  $A \cap A' = \phi$
- c) both (a) & (b)
- d) None of these.
- v) In the following graph

 $deg(V_3)$  is

a) 1

b) 0

c) 2

- d) 5.
- vi) If A and B are two subsets, then A and B are said to be disjoint if
  - a)  $A \cap B = \emptyset$
- b)  $A \cup B = \phi$
- c)  $A B = \phi$
- d) none of these.
- vii) If a set  $S = \{1, 2, 3\}$ , then the power set of S is
  - a)  $\{\phi, S\}$

b) { φ }

c) { S }

d) none of these.

a) 72

b) 144

c) 360

- d) None of these.
- ix) In a binary tree, the parent may have
  - a) right child
  - b) left child
  - c) both right and left childs
  - d) right or left or both childs.
- x) The Fuzzy logic is based on mapping the universe of discourse to
  - a) [0, 1]

b) (0, 1)

c)  $\{0, 1\}$ 

- d) none of these.
- xi) In Prime's Algorithm, the weight of non-existing edge is taken as
  - a) 0

b) + ∞

c) 1

- d) none of these.
- xii) Let L be a language given by  $L = \{a^n b^n : n \ge 0\}$ , then  $L^2$  is equal to
  - a)  $\{a^n b^n a^m b^m : n \ge 0, m \ge 0\}$
  - b)  $\left\{a^n b^n : n \ge 0\right\}$
  - c)  $\left\{a^n b^n a^m b^m : n \ge 0\right\}$
  - d) none of these.



- a)  $e \ge n + k$
- b)  $e \ge n k$
- c)  $e \le n k$
- d) none of these.

# GROUP – B ( Short Answer Type Questions )

Answer any *three* of the following.  $3 \times 5 = 15$ 

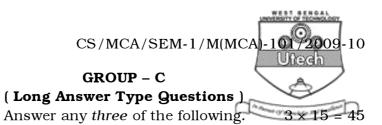
- 2. Consider the language  $L = \{0^n \ 1^n : n \neq m\}$ , find a context free grammar G which generates L.
- 3. Show that the maximum number of edges in a simple graph with n vertices is n (n-1) / 2.
- 4. Let A be some fixed 10-element subset of  $S = \{1, 2, 3, 4, 5, \dots, 50\}$ . Show that A possesses two different 5-element subsets, the sums of whose elements are equal.
- 5. Solve the following using generating function:

$$a_n - a_{n-1} = 3(n-1), n \ge 1$$
, and where  $a_0 = 2$ .

6. Find the coefficient of  $x^{18}$  in

$$(x + x^2 + x^3 + x^4 + x^5) (x^2 + x^3 + x^4 + x^5 + \dots)^5$$

- 7. Obtain equivalent disjunctive normal form of  $\sim G \land (H \Leftrightarrow G)$ .
- 8. Design a finite state machine that performs serial addition.



9. a) Let  $X = \{1, 2, 3, \dots, 7\}$  and

 $R = \{ (x, y) : x - y \text{ is divisible by } 3 \}$ . Prove that R is an equivalence relation and draw the relation graph.

b) Find the transitive closure of a relation R on the set  $\{a, b, c\}$ , whose relation matrix  $M_R$  is given as

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}.$$
 7 + 8

- 10. a) Prove that 21 divides  $4^{n+1} + 5^{2n-1}, \forall n > 0$ .
  - b) Let M be the finite state machine with state table appearing in the following table:

		f			g	
S A	a	b	С	а	b	С
$S_0$	$S_0$	$S_0$	$S_0$	0	1	0
$S_1$	$S_0$	$S_0$	$S_0$	1	1	1
$S_2$	$S_0$	$S_0$	$S_0$	1	0	0

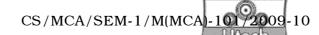
- i) Find the input set A, the state set S, the output set O, and initial state of M.
- ii) Draw the state diagram of M.

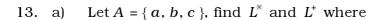
Find the output string for the input string *aabbcc*.

5 + 10

- 11. a) Prove that if there is one and only path between every pair of vertices in a graph G, then G is a tree.
  - b) Describe Kruskal's algorithm to find the Minimal spanning tree in a graph G. Use this algorithm to find minimal spanning tree for the following graph:

- c) Prove that a simple graph with n vertices and k components cannot have more than  $\frac{(n-k)(n-k+1)}{2}$  edges. 5+5+5
- 12. a) Prove that a simple graph has a spanning tree iff it is connected.
  - b) Find the sequence  $\{y_x\}$  having the generating function G, given by  $G(x) = \frac{3}{1-x} + \frac{1}{1-2x}$ .
  - c) By mathematical induction prove that  $3^{2n+1} + (-1)^n \ 2 \equiv 0 \ (\text{mod } 5).$  5 + 5 + 5





i) 
$$L = \{b^2\}$$

ii) 
$$L = \{a, b\}$$

b) Prove the following identities:

i) 
$$\lambda + 1^* (O11)^* (1^* (O11))^* = (1 + O11)^*$$

ii) 
$$(1+00*1)+(1+00*1)(0+10*1)*(0+10*1)=0*1(0+10*1)*$$

c) Draw the transition diagram of the non-deterministic finite-state automaton whose next state is given below:

A S	0	1
$S_0$	$\left\{ \boldsymbol{S}_{0},\boldsymbol{S}_{1}\right\}$	$\{\mathbf{S}_2\}$
$S_1$	Φ	$\{S_1\}$
$S_2$	$\left\{ \mathbf{S}_{1},\mathbf{S}_{2}\right\}$	Φ

5 + 5 + 5

14. a) Show that  $(p \lor q) \land (-p \land \sim q)$  is a contradiction.

b) Show that  $R \land (P \lor Q)$  is a valid conclusion from the premises  $P \lor Q, Q \Rightarrow R, P \Rightarrow M$  and  $\sim M$ .

c) Determine a DFA from the NDFA  $M = \left( \{q_0, \ q_1\}, \ \{0, \ 1\}, \ \delta, \ q_0, \ \{q_1\} \right), \text{ with the state transition }$ 

function  $\delta$  as given in the following table :

States	Input		
$\rightarrow q_0$	$\{oldsymbol{q}_0,oldsymbol{q}_1\}$	$\{oldsymbol{q}_1\}$	
$q_{\scriptscriptstyle 1}$ ( Final state )	Φ	$\{oldsymbol{q}_0,oldsymbol{q}_1\}$	

5 + 5 + 5

- 15. a) Prove that a simple graph G ( V, E ) has a spanning tree iff G ( V, E ) is connected graph.
  - b) Define the following by example:
    - i) DFA
    - ii) NDFA
  - c) If  $(A, \leq)$  and  $(B, \leq)$  are posets, then prove that  $\{(A \times B, \leq)\}$  is a poset with partial order  $\leq$  defined as  $(a, b) \leq (a, b)$ , if  $a \leq a$  in A and  $b \leq b$  in B. 5 + 5 + 5

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