

Name :

Roll No. :

Invigilator's Signature :

CS/MCA/SEM-1/M(MCA)-101/2009-10

2009

DISCRETE MATHEMATICAL STRUCTURES

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP - A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10

i) The number of arrangements of 25 objects where 7 are of the first kind, 12 are of the second kind, 3 are of the third kind and 4 are of the fourth kind is given by

a) $\frac{25!}{7!2!3!4!}$

b) $\frac{25!}{7!2!}$

c) $\frac{25!}{3!4!}$

d) none of these.

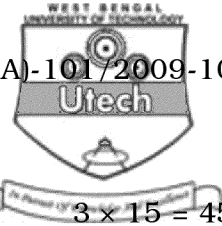
ii) The coefficient of X^{25} in $(X^3 + X^4 + X^5 + \dots)^5$ is

a) $C(9, 5)$

b) $C(5, 9)$

c) $C(5, 5)$

d) $C(9, 9)$



GROUP – C
(Long Answer Type Questions)

Answer any *three* of the following. 3 × 15 = 45

9. a) Let $X = \{ 1, 2, 3, \dots, 7 \}$ and
 $R = \{ (x, y) : x - y \text{ is divisible by } 3 \}$. Prove that R is an equivalence relation and draw the relation graph.

b) Find the transitive closure of a relation R on the set $\{ a, b, c \}$, whose relation matrix M_R is given as

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}. \quad 7 + 8$$

10. a) Prove that 21 divides $4^{n+1} + 5^{2n-1}, \forall n > 0$.

b) Let M be the finite state machine with state table appearing in the following table :

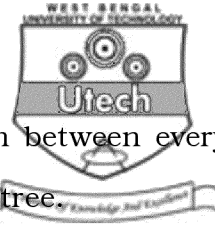
| | | f | | | g | | |
|---|-------|-------|-------|-------|---|---|---|
| | | a | b | c | a | b | c |
| S | A | | | | | | |
| | S_0 | S_0 | S_0 | S_0 | 0 | 1 | 0 |
| | S_1 | S_0 | S_0 | S_0 | 1 | 1 | 1 |
| | S_2 | S_0 | S_0 | S_0 | 1 | 0 | 0 |

i) Find the input set A , the state set S , the output set O , and initial state of M .

ii) Draw the state diagram of M .

Find the output string for the input string $aabbcc$.

5 + 10



11. a) Prove that if there is one and only path between every pair of vertices in a graph G , then G is a tree.

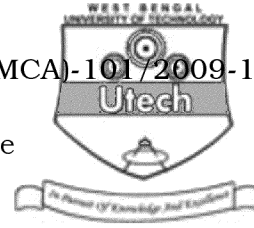
b) Describe Kruskal's algorithm to find the Minimal spanning tree in a graph G . Use this algorithm to find minimal spanning tree for the following graph :

c) Prove that a simple graph with n vertices and k components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges. 5 + 5 + 5

12. a) Prove that a simple graph has a spanning tree iff it is connected.

b) Find the sequence $\{y_x\}$ having the generating function G , given by $G(x) = \frac{3}{1-x} + \frac{1}{1-2x}$.

c) By mathematical induction prove that $3^{2n+1} + (-1)^n 2 \equiv 0 \pmod{5}$. 5 + 5 + 5



13. a) Let $A = \{ a, b, c \}$, find L^* and L^+ where

i) $L = \{ b^2 \}$

ii) $L = \{ a, b \}$

b) Prove the following identities :

i) $\lambda + 1^* (011)^* (1^* (011))^* = (1 + 011)^*$

ii) $(1 + 00^* 1) + (1 + 00^* 1)(0 + 10^* 1)^* (0 + 10^* 1) = 0^* 1(0 + 10^* 1)^*$

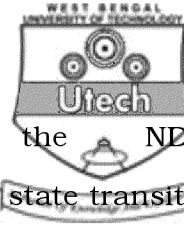
c) Draw the transition diagram of the non-deterministic finite-state automaton whose next state is given below :

| S \ A | 0 | 1 |
|-------|------------------|-------------|
| S_0 | $\{ S_0, S_1 \}$ | $\{ S_2 \}$ |
| S_1 | Φ | $\{ S_1 \}$ |
| S_2 | $\{ S_1, S_2 \}$ | Φ |

5 + 5 + 5

14. a) Show that $(p \vee q) \wedge (\sim p \wedge \sim q)$ is a contradiction.

b) Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q, Q \Rightarrow R, P \Rightarrow M$ and $\sim M$.



- c) Determine a DFA from the NFA $M = (\{q_0, q_1\}, \{0, 1\}, \delta, q_0, \{q_1\})$, with the state transition function δ as given in the following table :

| States | Input | |
|-----------------------|----------------|----------------|
| $\rightarrow q_0$ | $\{q_0, q_1\}$ | $\{q_1\}$ |
| q_1 (Final state) | Φ | $\{q_0, q_1\}$ |

5 + 5 + 5

15. a) Prove that a simple graph $G (V, E)$ has a spanning tree iff $G (V, E)$ is connected graph.
- b) Define the following by example :
- i) DFA
 - ii) NFA
- c) If (A, \le) and (B, \le) are posets, then prove that $\{(A \times B, \le)\}$ is a poset with partial order \le defined as $(a, b) \le (a, b)$, if $a \le a$ in A and $b \le b$ in B . 5 + 5 + 5

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