Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/MCA/SEM-1/M(MCA)-101/2010-11

2010-11
DISCRETE MATHEMATICAL STRUCTURES

Time Allotted : 3 Hours | Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) A relation R is called an equivalence relation if

      a) R is reflexive and transitive

      b) R is reflexive and symmetric

      c) R is reflexive, transitive and symmetric

      d) R is reflexive, anti-symmetric and transitive.

   ii) If A and B are nonempty sets. Then cardinality of A and B are 2 & 3 respectively then cardinality of $A \times B$ is

      a) 6 b) 5

      c) 13 d) 4.

   iii) The coefficient of $x^3y^2z^2$ in $(x+y+z)^9$ is

      a) (9!)/(7!) b) (9!)/(3! 2! 2!)

      c) (7!)/(7!) d) (3! 2! 2!)/(7!).

iv) The membership function of a fuzzy set is defined in

a) { 0, 1 } b) (0, 1)

c) [0, 1] d) none of these.

v) The maximum number of vertices in a connected graph having 17 edges is

a) 18 b) 17

c) 19 d) 12.

vi) Let L be a language given by $L = \{a^n b^n : n \geq 0\}$ then L^2 is equal to

a) $\{a^n b^n a^m b^m : n \geq 0,\ m \geq 0\}$

b) $\{a^n b^n : n \geq 0\}$

c) $\{a^n b^n a^m b^m : n \geq 0\}$

d) none of these.

vii) In Prime's Algorithm, the weight of non-existing edge is take as

a) 0 b) 1

c) + infinity d) none of these.

viii) There are 4 blue, 3 red and 2 black pens in a box. These are drawn one by one. Determine all the different permutations.

a) 1080 b) 1060

c) 1100 d) None of these.

ix) Every self complementary graph has vertices

a) $4k + 1$ b) $4k$

c) $4k + 1$ or $4k$ d) none of these.

x) The proposition $P \wedge (\sim p \vee q)$ is

a) a tautology

b) logically equivalent to $p \wedge q$

c) logically equivalent to $p \vee q$

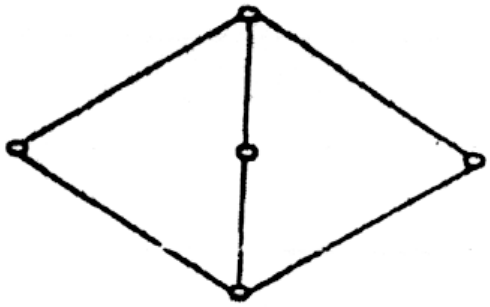
d) a contradiction.

xi) Which of the following is/are tautology ?

a) $a \vee b \rightarrow b \wedge c$ b) $a \wedge b \rightarrow b \vee c$

c) $a \vee b \rightarrow (b \rightarrow c)$ d) $a \rightarrow b \rightarrow (b \rightarrow c)$.

xii) The following is the Hass diagram of the poset $|\{a, b, c, d, e\}|$. The poset is



a) not a lattice

b) a lattice but not a distributive lattice

c) a distributive lattices but not a Boolean algebra

d) a Boolean algebra.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. 3 × 5 = 15

2. Prove that for a simple graph having n vertices and k components, the maximum number of edges is $\frac{(n-k)(n-k+1)}{2}$.

3. Prove that in a bounded distributive lattice, complement of an element is unique.

4. Obtain a conjunctive normal form of $p \wedge (p \Rightarrow q)$.

5. Design a finite state machine that performs serial addition.

6. Draw the transition diagram for the FSA with $I = \{ a, b \}$, $Q = \{q_0, q_1, q_2\}$, $F = \{q_0, q_1\}$ and δ is given by

δ	a	b
q_0	q_0	q_1
q_1	q_0	q_2
q_2	q_2	q_2

GROUP – C

(Long Answer Type Questions)

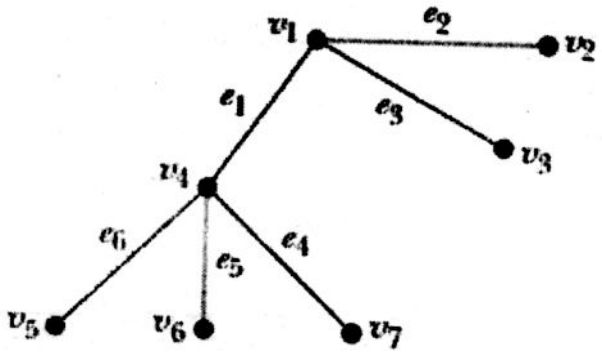
Answer any *three* of the following. 3 × 15 = 45

7. a) In a set A, a relation R is defined as follows

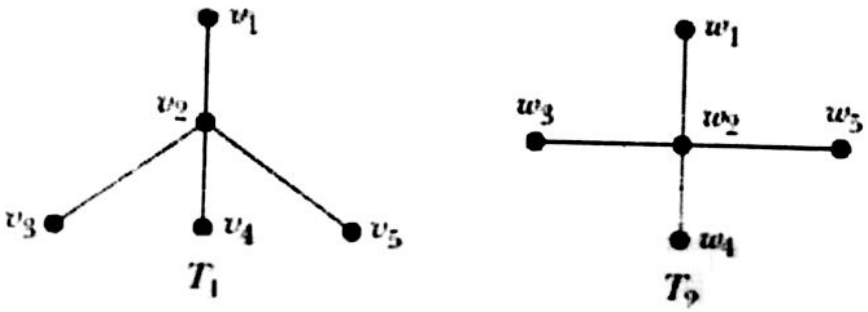
$$R = \{(a_1, a_1),(a_1, a_2),(a_1, a_4),(a_2, a_3),(a_3, a_3),$$
$$(a_3, a_5)(a_4, a_4)(a_5, a_2)\}$$

Determine the transitive closure of R using Warshall's algorithm.

b) Determine the adjacency matrix for the following graph.



c) Examine whether the following trees are isomorphic



9 + 3 + 3

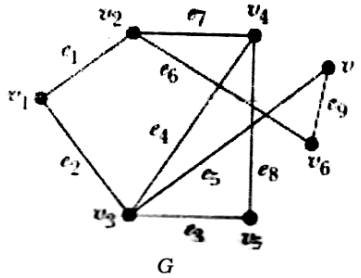
8. a) Evaluate the following postfix expression

24 4 + 3·7/26 4 – +

b) Draw the tree for the following infix expression and find the corresponding prefix expression

(($A + B$)/($C – D$) + E). $F – G$.

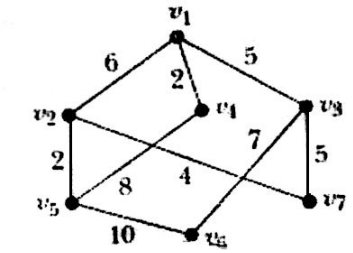
c) Draw a spanning tree of the following graph :



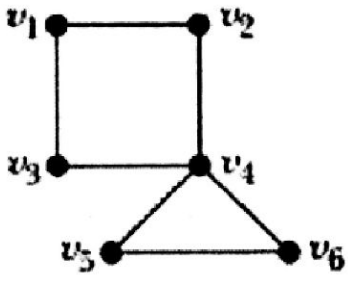
G

4 + 8 + 3

9. a) Apply Prim's algorithm to find a minimum spanning tree from the following weighted graph.



b) Apply BFS algorithm to find a spanning tree of the following graph



c) Draw the transition diagram from the DFA given by

$M = (\{q_0, q_1, q_2\}, \{0, 1\}, q_0, \delta, \{q_2\})$

$\delta(q_0, 0) = q_1$, $\delta(q_1, 1) = q_2$,

$\delta(q_0, 1) = q_0$, $\delta(q_2, 0) = q_0$,

$\delta(q_1, 0) = q_2$, $\delta(q_2, 1) = q_1$, 5 + 5 + 5

10. a) What is Language and what is Grammar ? Why Language & Grammar is needed for computer science ?

b) Construct the grammar for the language $L = a^n b^n c^m d^m$, $m, n, > 0$.

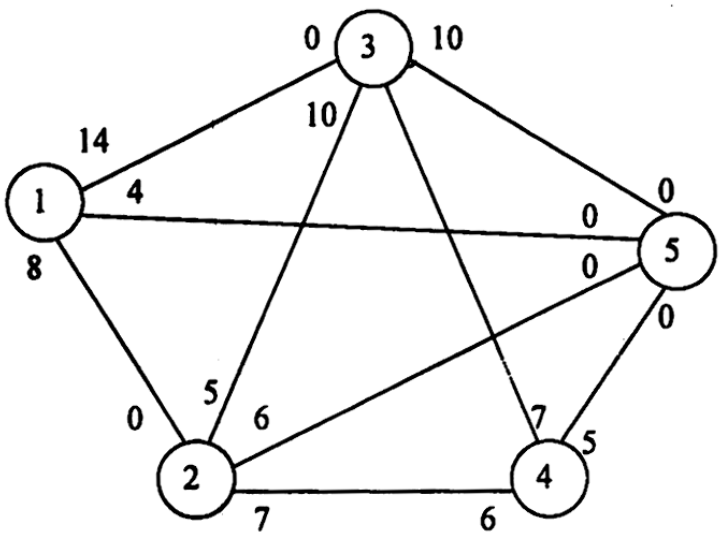
c) Construct a Moore machine equivalent to the Mealy machine M given by the following table :

Present state	Next State			
	$a = 0$		$a = 1$	
	state	output	state	output
$-> q1$	$q1$	1	$q2$	0
$q2$	$q4$	1	$q4$	1
$q3$	$q2$	1	$q3$	1
$q4$	$q3$	0	$q1$	1

d) Construct a Mealy machine which is equivalent to the Moore machine given by the following table :

Present state	Next State		Output
	$a = 0$	$a = 1$	
$-> q0$	$q1$	$q2$	1
$q1$	$q3$	$q2$	0
$q2$	$q2$	$q1$	1
$q3$	$q0$	$q3$	1

11. a) Determine the maximal for in the following network :



b) Show that $f : R \rightarrow (-1, 1)$ given by $f(x) = \frac{x}{1+|x|}$ is injective.

c) Prove that in a simple graph with n vertices and m components can have at most $\frac{(n-m)(n-m+1)}{2}$ edges. 7 + 2 + 6