MM-301

STATISTICS AND NUMERICAL TECHNIQUES

Time Allotted: 3 Hours | Full Marks: 70

The questions are of equal value.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP A
(Multiple Choice Type Questions)

1. Answer any *ten* questions. $10\times1 = 10$

(i) The number of significant digits in 1.00234 is

(A) 4 (B) 6 (C) 3 (D) 5

(ii) The percentage error in approximating 4/3 to 1.3333 is

(A) 0.0025% (B) 25% (C) 0.00025% (D) 0.25%

(iii) If E_a is the absolute error in a quantity whose true and approximate values are given by x_t and x_a then relative error is given by

(A) $\left|\frac{E_a}{x_a}\right|$ (B) $\left|\frac{E_a}{x_t}\right|$ (C) $\left|\frac{E_a}{x_t - x_a}\right|$ (D) $|E_a|$

(iv) The probability that a leap year selected at random will contain 53 Wednesdays is

(A) $\frac{53}{366}$ (B) $\frac{365}{366}$ (C) $\frac{2}{7}$ (D) $\frac{5}{7}$

(v) The condition for two events A and B to be independent is

(A) $P(A \cap B) = P(A)\,P(B)$ (B) $P(A + B) = P(A)\,P(B)$

(C) $P(A - B) = P(A)\,P(B)$ (D) $P(A \cap B)\,P(A)\;P(B|A)$

(vi) If $P(A \cap B) = \frac{1}{2}$, $P(A^C \cap B^C) = \frac{1}{3}$, $P(A) = P(B) = p$, then p is

(A) $\frac{7}{12}$ (B) $\frac{5}{6}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$

(vii) The n^{th} order forward difference of the n^{th} degree polynomial is

(A) $n!$ (B) $(n+1)!$ (C) 0 (D) none of these

(viii) In Simpson's 1/3 rule of finding $\int_a^b f(x)dx$, $f(x)$ is approximated by

(A) line segment (B) parabola (C) circular sector (D) part of ellipse

(ix) The error involved in 4^{th} order R-K method is given by

(A) $O(h^2)$ (B) $O(h^4)$ (C) $O(h^3)$ (D) $O(h^5)$

(x) Which one of the following is not true?

(A) $\Delta = E - 1$ (B) $\Delta \cdot \nabla = \Delta - \nabla$ (C) $\Delta / \nabla = \Delta + \nabla$ (D) $\nabla = 1 - E^{-1}$

(xi) Which of the following is an iterative method?

(A) Gauss elimination method (B) Gauss Jordon method
(C) LU decomposition method (D) Gauss-Seidel method

(xii) $(\Delta - \nabla)x^2 =$

(A) h^2 (B) $-2h^2$ (C) $2h^2$ (D) none of these

GROUP B
(Short Answer Type Questions)

Answer any *three* questions. 3×5 = 15

2. Find the mean, variance and standard deviation of the Binomial distribution with parameters n and p.

3. The p.d.f of a continuous distribution of a random variable x is given by
$f(x) = ke^{-2x}$, $x > 0$
$= 0$ elsewhere.
Find the value k, and distribution function $F(x)$.

4. Using Euler's method obtain the solution of $\frac{dy}{dx} = x - y$ with $y(0) = 1$ and $h = 0.2$ at $x = 0.4$

5. Apply Trapezoidal rule to evaluate $\int_0^1 \frac{dx}{(x^2 - 2x + 2)^3}$, by taking 5 equal subintervals.

6. Apply Gaussian elimination method to solve

$$\begin{pmatrix} 3 & 4 & 5 \\ 2 & -3 & 4 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 40 \\ 13 \\ 9 \end{pmatrix}$$

GROUP C
(Long Answer Type Questions)

Answer any *three* questions. $3 \times 15 = 45$

7. (a) There are two boxes, the first box containing 3 white and 7 black balls and the second containing 7 white and 3 black balls. One box is chosen at random and from it 2 balls are drawn without replacement. Find the probability that both the balls are white. Also, given that both the balls are white, find the conditional probability that the first ball was chosen. 5

(b) If X is normally distributed with mean 3 and s.d. 2, find c such that $P(X > c) = 2P(X \leq c)$. Given that $\int_{-\infty}^{0.43} \Phi(z)\,dz = 0.666$. 5

(c) The mean and s.d. of 20 items are found to be 10 and 2 respectively. At the time of checking it was found that one item 8 was incorrect. Calculate the mean and S.D. if 5

(i) the wrong item is omitted

(ii) it is replaced by 2

8. (a) Solve by LU decomposition method 8

$$x + y + z = 6$$
$$x + 2y + 3z = 14$$
$$x - y + z = 2$$

(b) Find a root of $x^3 + 2x - 2 = 0$ by Regula-Falsi method, correct up to three significant figure. 7

9. (a) Apply Newton interpolation formula to find $y(1.5)$ and $y(4.5)$ from the following data: 8

X:	0	1	2	3	4	5
Y:	1	5	31	121	341	781

(b) if 7

x	4.0	4.2	4.4	4.6	4.8	5.0	5.2
$f(x)$	1.3883	1.4351	1.4876	1.5261	1.5686	1.6094	1.6487

find $\int_{4.0}^{5.2} f(x)dx$ by Weddle's rule.

10.(a) Apply Taylor series method to determine $y(0.1)$ by solving the equation $\frac{dy}{dx} = 3x + y^2$; $y(0) = 1$. 4

(b) Apply Newton Raphson method to find a root of $f(x) = \sin x - 3x + 1 = 0$ 5

(c) If the equations of two lines be $3x + 12y = 19$ and $3y + 9x = 46$, determine which one of these is the regression equation of y on x and which one of these is the regression equation of x on y. Give reasons. Also find coefficient of correlation. 6

11.(a) Compute $y(1.4)$ by Milne's predictor and corrector's method from $\frac{dy}{dx} = \frac{1}{2}(x + y)$ where $y(1) = 3.595$, $y(1.1) = 3.833$, $y(1.2) = 4.088$, $y(1.3) = 4.362$ 5

(b) Evaluate $\int_1^3 \frac{xdx}{x^2+3}$ by Romberg integration method. 5

(c) Find the negative root of the equation $x^2 - 0.7x - 2.5 = 0$, correct up to 2 decimal places using Bisection method. 5