Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/MCA/SEM-3/M(MCA)-301/2011-12

2011

STATISTICS AND NUMERICAL TECHNIQUES

Time Allotted : 3 Hours Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

$10 \times 1 = 10$

i) E^{-1} is equivalent to

a) $1 - \nabla$ b) $1 + \Delta$

c) $1 - \nabla$ d) none of these.

ii) if $Var\,(aX + bY) = a^2\ Var\,(X) + b^2\ Var\,(Y)$, the X and Y are

a) mutually exclusive b) uncorrelated

c) impossible events d) none of these.

iii) If E^c is the complement of the event E then

a) $P(E^c) = 1 - P(E)$

b) $P(E^c) = P(E)$

c) $P(E^c) = 1 + P(E)$

d) none of these.

iv) For a binomial distribution

a) Mean > Variance b) Mean = Variance

c) Mean < Variance d) none of these.

v) If $f(x)$ is a polynomial of degree n, then $\Delta^n f(x)$ is

a) 0 b) constant

c) 1 d) none of these.

vi) Romberg's method is based on the error in

a) Trapezoidal rule

b) Simpson's 1/3rd rule

c) Weddle's rule

d) none of these.

vii) Order of convergence of Newton-Raphson method is

a) 0 b) 2

c) 1 d) none of these.

viii) Normal distribution is

a) unimodal b) bimodal

c) trimodal d) none of these.

ix) Order of error in Simpson's 1/3rd rule is

a) h^2 b) h^4

c) 1 d) none of these.

x) Condition for convergence of Fixed-point iteration method to solve the equation $f(x) = 0$ in $[a, b]$ is that

a) $\phi'(x) < 1$ b) $\phi'(x) > 1$

c) $\phi'(x) = 1$ d) $\phi'(x) \leq 1$

in $[a, b]$ where $f(x) = 0$ can be written as $\phi(x) = x$.

xi) The formula of the area of a trapezium whose length of the parallel sides are a, b and the distance between them is h is

a) $\frac{h}{2}(a + b)$ b) $\frac{h}{2} + a + b$

c) $h(a + b)$ d) $h + a + b$.

xii) The method of bisection for solving equation $f(x) = 0$ in $[a, b]$ is based on

a) Intermediate value theorem

b) MVT of integral calculus

c) MVT of differential calculus

d) Fundamental theorem of Algebra.

xiii) $\int_a^b f(x)\,dx$ describe the

a) area

b) volume

c) surface area

d) volume and surface area both

under the curve $y = f(x)$ in $[a, b]$.

xiv) In Newton's forward and backward interpolation formula the points are

a) equally spaced

b) unequally spaced

c) both of the previous

d) none of the previous.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Prove that $P(A \cup B) = P(A) + P(B)$, if A and B are disjoint events.

3. The probabilities of X, Y and Z being managers are in the ratio 4 : 2 : 3 respectively. The probabilities that the bonus scheme will be introduced if X, Y, Z become managers are $\frac{3}{10}$, $\frac{1}{2}$, $\frac{4}{5}$ respectively.

i) What is the probability that bonus scheme will be introduced ?

ii) If the bonus scheme has been introduced, what is the probability that the manager appointed was Y ?

4. Evaluate $\int_0^1 x/(1+x)\,dx$ using Trapezoidal rule using 5 intervals.

5. Evaluate $\int_1^2 \log x\,dx$ using Simpson's 1/3rd rule using 5 intervals.

6. Distinguish between absolute error and relative error with example.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Use Newton-Raphson method to find a positive root of $e^x = 3x$ correct to four decimal places.

   b) What are the advantages and disadvantages of Newton-Raphson method ?

   c) State and prove Bayes' theorem. 6 + 4 + 5

8. a) Find $\Delta^2 f(x)$ where $f(x) = 3x^4 + 8x^2 + 5x + 7$ by taking $h = 1$.

   b) Apply Simpson's 1/3rd rule to find $\int_0^{\pi/2} \cos x\,dx$ by dividing the range on integration into 6 subintervals.

   c) Prove that if E_1 and E_2 are statistically independent, then $P(E_1 \cap E_2) = P(E_1)\,P(E_2)$. 6 + 6 + 3

9. a) Discuss the convergence of fixed point iteration.

b) Prove that if ρ_{xy} is the Pearson correlation coefficient between the random variables X and Y, then $-1 \leq \rho_{xy} \leq 1$.

c) Apply Newton's forward interpolation to find $f(x)$ at $x = 2{\cdot}5$ from the following table :

x	2	3	4	5	6
$f(x)$	1·456	1·689	1·992	2·010	2·225

5 + 5 + 5

10. a) Find $\sqrt{45}$ usng Newton-Raphson method.

b) Use Gauss-Jordan method to solve

$p + 2q + r - s = -2$

$2p + 3q - r + 2s = 7$

$p + q + 3r - 2s = -6$

$p + q + r + x = 2$.

c) Prove that if $X \sim$ Binomial (n, p) then $E(X) = np$.

5 + 5 + 5

11. a) Derive the expression of error in the composite trapezoidal rule.

b) Apply Runge-Kutta method of order 4 to solve $\frac{dy}{dx} = x + y$, where $y(0) = 1$ at $x = 0{\cdot}1$ and $0{\cdot}2$. 7 + 8

12. a) Apply LU factorization to find the inverse of

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$$

b) Prove that for a normal distribution :

Mean = Median = Mode

c) Fit an approximating polynomial to the following data :

x	0	3	4
$f(x)$	2·12	4·34	3·19

8 + 3 + 4