http://www.makaut.com

WEST BENGAL UNIVERSITY OF TECHNOLOGY

MM-401

OPERATION RESEARCH AND OPTIMIZATION TECHNIQUES

Time Allotted: 3 Hours | Full Marks: 70

The questions are of equal value.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

GROUP A
(Multiple Choice Type Questions)

1. Answer any *ten* questions. $10 \times 1 = 10$

(i) Given a system of m simultaneous equations in n unknowns ($m < n$), the number of basic variables will be

(A) m (B) n
(C) m – n (D) m + n

(ii) Which of the following statements regarding artificial variables are true?

(A) the artificial variable is introduced to get an identity matrix
(B) the artificial variables have no real physical significance
(C) all the above
(D) only (B) but not (A)

(iii) The role of artificial variable in the simplex method is

(A) to aid in finding an initial solution
(B) to find optimal dual price in the final simplex table
(C) to start phases of simplex method
(D) all of these

http://www.makaut.com

http://www.makaut.com

(iv) Every extreme point of the convex set of all feasible solutions of the system $Ax = b, x \geq 0$ corresponds to

(A) a basic solution
(B) a feasible solution
(C) both (A) and (B)
(D) only (B) but not (A)

(v) Which one of the following is not a deterministic method

(A) L.P.P.
(B) T.P.
(C) C.P.M.
(D) P.E.R.T.

(vi) If there are n workers and n job, there would be

(A) n! solutions
(B) (n –1)! solutions
(C) $(n!)^n$ solutions
(D) n solutions

(vii) Let the time estimates for a particular activity be t_o = 5 days, t_m = 7 days, t_p = 9 days. Then the expected time t_e is (where the symbols have their usual meanings)

(A) 10 days
(B) 15 days
(C) 5 days
(D) 7 days

(viii) The balance transportation problem is where

(A) Total supply > Total demand
(B) Total supply < Total demand
(C) Total supply = Total demand
(D) None of these

(ix) In a simple deterministic EOQ model, with constant demand rate (D) and infinite rate of production, the economic lot size is

(A) $\sqrt{2K/Dh}$
(B) $\sqrt{2/KDh}$
(C) $\sqrt{2KDh}$
(D) $\sqrt{2KD/h}$

(x) In an assignment problem, the minimum number of lines covering all the zeros in the reduced cost matrix of order n can be

(A) at most n
(B) n + 1
(C) n – 1
(D) at least n

http://www.makaut.com

http://www.makaut.com

(xi) Consider the unit simplex $S = \{ (x,y,z) : x + y + z \leq 1, x,y,z \geq 0\}$

Then the number of vertices S has

(A) 2 (B) 4

(C) 5 (D) 7

(xii) If the primal L.P.P. has degenerate optimal solution, then the dual has

(A) alternate optimal solution (B) degenerate optimal solution

(C) no feasible solution (D) no optimal solution

GROUP B
(Short Answer Type Questions)

Answer any *three* questions. $3 \times 5 = 15$

2. Solve the following LPP by graphical method and then find the optimal solution, if exists,

Minimize $Z = x_1 + x_2$.

Subject to $5x_1 + 9x_2 \leq 45$;

$x_1 + x_2 \geq 2$;

$x_2 \leq 4$

and $x_1, x_2 \geq 0$.

3. Find the dual of the following L.P.P.

Minimize $Z = 2x_1 + 7x_2 + 5x_3$

Subject to $2x_1 + 5x_2 + 7x_3 \leq 17$

$3x_1 + 2x_2 + 5x_3 = 13$

$5x_1 + 3x_2 + x_3 \leq 9$

and $x_1, x_3 \geq 0$, x_2 unrestricted in sign.

4. Use simplex method to solve the following problem:

Maximize $Z = 2x_1 + 5x_2$.

Subject to, $x_1 + 4x_2 \leq 24$; $3x_1 + x_2 \leq 21$; $x_1 + x_2 \leq 9$.

and $x_1, x_2 \geq 0$.

http://www.makaut.com

http://www.makaut.com

5. For the game with payoff matrix:

Player	B_1	B_2	B_3
A_1	–1	2	–2
A_2	6	4	–6

Determine the optimal strategies for players A and B. Also determine the values of game.

6. The production department for a company requires 3600 kg of a raw material for manufacturing a particular item per year. It has been estimated that the cost of placing an order is Rs. 36 and the cost of carrying inventory is Rs. 25 percent of the investment in the inventories. The price is Rs. 10 per kg. Find the minimum total annual inventory cost.

GROUP C
(Long Answer Type Questions)

Answer any *three* questions. $3 \times 15 = 45$

7. (a) Show that $X = \{(x_1,x_2) : 9x_1^2 + 4x_2^2 \leq 36\}$ is a convex set. 5

(b) Using Simplex method Solve the following LPP: 8

Minimize $Z = 2x_1 + 3x_2$

Subject to $x_1 + x_2 \leq 8$

$x_1 + 2x_2 = 5$

$2x_1 + x_2 \leq 8$

and $x_1 , x_2 \geq 0$ by Charnes Big M method.

(c) Find the basic feasible solutions of the system of equations 2

$x_1 + x_2 + x_3 = 8$

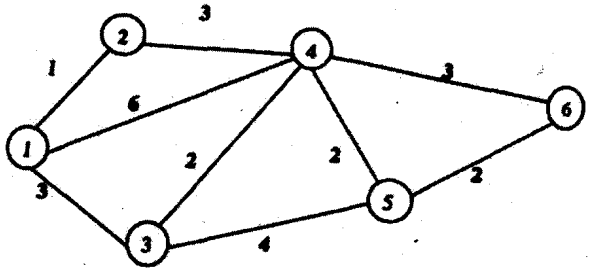
$3x_1 + 2x_2 = 18$

8. (a) In Birth and Death model show that the expected number of customers in the system is $\rho/(1-\rho)$, where $\rho = \lambda/\mu$, λ= mean arrival rate of customer, μ = mean service rate of the customer. 7

http://www.makaut.com

http://www.makaut.com

(b) Using Dijkstra's algorithm find the shortest path and the length (or weight) of the shortest path of the following network between node 1 to node 6. 8



9. (a) Consider the game G whose pay-off matrix is 5

	Player B	
Player A	2	6
	–2	μ

(i) Prove that the game is strictly determinable for any value of μ
(ii) Find out the value of G.

(b) Find an optimal basic feasible solution of the following transportation problem and the optimum transportation schedule and minimum transportation cost. 10

	D_1	D_2	D_3	D_4	Available
O_1	19	30	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
Demand	5	8	7	14	

10.(a) A machine operator has to perform three operations, turning threading and knurling, on a number of different jobs. The time required to perform these operations (in minutes) on each job is known. Determine the order in which the jobs should be processed in order to minimize the total time required to turn out all the jobs. Also find the minimum elapsed time. 7

Job	A	B	C	D	E	F	G
Turning	3	8	7	4	9	8	7
Threading	4	3	2	5	1	4	3
Knurling	6	7	5	11	5	6	12

http://www.makaut.com

http://www.makaut.com

(b) Use dynamic programming to solve 8

Minimize $Z = Y_1^2 + Y_2^2 + Y_3^2$

Subject to the constraints $Y_1 + Y_2 + Y_3 \geq 15$

And $Y_1, Y_2, Y_3 \geq 0$

11.(a) Draw the network and find the critical path for the following activities of a project 9

Activity	A	B	C	D	E	F
Immediate predecessor	–	A	A	B,C	–	E
Duration (Days)	2	3	4	6	2	8

(b) If the arrival rate is λ and the service rate is μ, then prove that the expected queue length is $\frac{\lambda^2}{\mu(\mu-\lambda)}$ 6

12.(a) Explain ABC, VED and FSN analysis with examples. 7+8

(b) The tool room company's quality control department is manned by a single clerk who takes an average of 5 minutes in checking parts of each of the machine coming for inspection. The machine arrives once in every 8 minutes on the average. One hour of the machine is valued at Rs. 15 and the clerk's time is valued at Rs. 4 per hour. What is the average hourly queuing system costs associated with the quality control department?

http://www.makaut.com