# 2014

## Operations Research and Optimization Techniques

*Time Alloted : 3 Hours* *Full Marks : 70*

***The figure in the margin indicate full marks.***
***Candidates are required to give their answers in their own words as far as practicable***

## GROUP - A
## ( Multiple Choice Type Questions )

**1. Choose the correct alternatives for any ten of the following:** **10x1=10**

i) **The solution to a transportation problem with m rows and n columns is feasible it number of positive allocations are**

a) m + n
b) mn
c) m + n-l
d) m + n+ 1

ii) **If x is a convex combination of y and z and x = ay + bz then**

a) a+b=1
b) a+b=0
c) a=b
d) none of these.

iii) **The possible no. of basic solutions in a system of equations**

with n unknown is

a) a) mn
b) m+n
c) $\frac{n!}{m!(n-m)!}$
d) none of these

iv) The shortest path between any two nodes in a Network is determined by the following

a) Dijkastra's algorithm
b) Floyd's algorithm
c) Critical path method
d) none of these

v) The queue length is given by

a) $\frac{p^2}{1-p}$
b) $\frac{p}{1-p}$
c) $\frac{p^3}{1-p}$
d) none of these

vi) A two-person zero-sum game is said to be fair if

a) both the players have equal number of strategies
b) the game has a saddle point
c) the game does not have a saddle point
d) the value of the game is zero.

vii) The formula for finding the minimum inventory cost under the purchasing model without shortage is

a) $\sqrt{2RC_1C_3}$
b) $\frac{\sqrt{2C_3R}}{C_1}$

c) $\frac{\sqrt{C_1}}{2RC_3}$

d) none of these

viii) A basic solution of the system of equations:

$2x_1 + x_2 - x_3 = 2,\ 3x_1 + 2x_2 - x_3 = 3$

a) (1,1,1) (b) (1,1,0)

c) (1,0,0) (d) none of these

ix) If the duel has an unbounded solution, then primal has

a) An unbounded solution

b) An infeasible solution

c) A feasible solution

d) None of them

x) Dynamic problem deals with the

a) Multi-stage decision making problems.

b) Single stage decision making problems.

c) Time dependent decision making problems.

d) Problems which fix the levels of different decision variables so as to maximize profit or minimize the loss.

xi) In PERT network, the starting vertex is a

a) burst node

b) merge node

c) root

d) none of these

xii) What is the method used to solve a LPP involving artificial variables

a) Simplex method

b) Charnes's Big M

c) VAM

d) none of these

## GROUP - B
## ( Short Answer Type Questions )

**Answer any *three* of the following. 3x5=15**

**2. Solve the following game whose pay-off matrix is given by**

$$\begin{array}{c} \\ Player\,A \end{array} \begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{bmatrix} 1 & 3 & 1 \\ 0 & -4 & -3 \\ 1 & 5 & -1 \end{bmatrix} \end{array} \quad Player\,B$$

**3. Obtain the dual of the following L.P.P :**

$$Min\ Z = x_1 - x_2 + 2x_3$$

**Subject to**

$x_1 + x_2 + 4x_3 \geq 7,$

$x_2 - 3x_3 \geq 10,$

$3x_1 + x_2 + x_3 \geq 3,$

$x_1, x_2, x_3 \geq 0.$

**4. A company has five jobs to be done. The adjacent matrix shows the return in rupees on assigning the machines. Assign the five jobs to five machines to maximize the expected return.**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| P | 32 | 38 | 40 | 28 | 40 |
| Q | 40 | 24 | 28 | 21 | 36 |
| R | 41 | 27 | 33 | 30 | 37 |
| S | 22 | 38 | 41 | 36 | 36 |
| T | 29 | 33 | 40 | 35 | 39 |

5. Solve the following L.P.P. by simplex method

Min $Z = -x_1 + x_2$

Subject to

$5x_1 + 9x_2 \leq 45$,

$x_1 + x_2 \geq 2$,

$x_2 \leq 4$, $x_1, x_2 \geq 0$.

6. Find the Initial Basic Feasible Solution and corresponding transportation cost of the following transportation problem by using VAM.

Destination

| Origins | | D1 | D2 | D3 | D4 | Available |
|---|---|---|---|---|---|---|
| | O1 | 11 | 13 | 17 | 14 | 250 |
| | O2 | 16 | 18 | 14 | 10 | 300 |
| | O3 | 21 | 24 | 13 | 10 | 400 |
| | Requirement | 200 | 225 | 275 | 250 | |

## GROUP - C
## ( Long Answer Type Questions )

Answer any *three* of the following. 3x15=45

7. (a) Use dynamic programming to find the value of

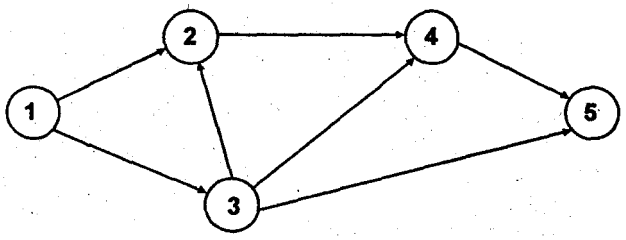
Max$Z = y_1 y_2 y_3$

Subject to the constraints:

$y_1 y_2 y_3 = 5$

$y_1, y_2, y_3 \geq 0$

b) Determine the Shortest path and length of the shortest Path by using Dijkstra's Algorithm of the following weighted network between sources 1 to sink 5.




8+7

8. (a) Establish EOQ model with uniform production rate , known demand, lead time zero and not shortage. Find the optimum order quantity and the optimum cost.

(b) Use two phase method to solve

Maximize $Z = 5x_1 + 3x_2$

Subject to $3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0$

8+7

9. a) A hospital has the following requirement of nurses :

| Period | Clock Time (24 hours day) | Minimum requirement |
|---|---|---|
| 1. | 6am-10am | 60 |
| 2. | 10am-2pm | 70 |
| 3. | 2pm-6pm | 60 |
| 4. | 6pm-10pm | 50 |
| 5. | 10pm-2am | 20 |
| 6. | 2am-6am | 30 |

Nurses report to the hospital at the beginning of each period and work for eight consecutive hours. The Hospital wants to determine the minimum number of nurses so that there may be sufficient number of nurses available for each period. Formulate

this as an L.P.P.

b) Find the optimal assignment for the assignment problem with the given cost matrix

| | a | b | c | d |
|---|---|---|---|---|
| A | 10 | 9 | 7 | 8 |
| B | 5 | 8 | 7 | 7 |
| C | 5 | 4 | 6 | 5 |
| D | 2 | 3 | 4 | 3 |

8+7

10. a) A firm manufactures products A and B and sells them at a profit of Rs. 2 and Rs. 3 respectively. Each product is processed on two machines I and II. Type A requires 1 minute processing on machine I and 2 minutes on machine II. Type B takes 1 minute in both machines. Machine I is available for not more than 6 hours and 40 minutes, while machine II is available is available not more than 10 hours in any working day. Formulate this L.P.P. mathematically and solve by graphical method.

(b) Solve the following LP problem using dual simplex method:

Maximize $Z = x_1 + x_2$

Subject to,

$3x_1 + 2x_2 \leq 5$

$x_2 \leq 2$

$x_1 x_2 \geq 0$

8+7

11. a) Solve the following assignment problem and find out the minimum total time.

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| $J_1$ | 3 | 5 | 10 | 15 | 8 |

| | | | | | |
|---|---|---|---|---|---|
| $J_2$ | 4 | 7 | 15 | 18 | 8 |
| $J_3$ | 8 | 12 | 20 | 20 | 12 |
| $J_4$ | 5 | 5 | 8 | 10 | 6 |
| $J_5$ | 10 | 10 | 15 | 25 | 10 |

b) Find the sequence that minimizes the total time required in performing the following jobs on three machines in the order ABC. Processing times (in hours) are given in the following table:

| Jobs | J1 | J2 | J3 | J4 | J5 |
|---|---|---|---|---|---|
| Machine A | 8 | 10 | 6 | 7 | 11 |
| Machine B | 5 | 6 | 2 | 3 | 4 |
| Machine C | 4 | 9 | 8 | 6 | 5 |

12. a) Show that in a pure birth process with mean arrival rate $\lambda$, the probability that there will be n arrivals in time is

$$p_n(t) = e^{-\lambda t}\frac{(\lambda t)^n}{n!}$$

b) The owner of a small machine shop has four machinists available to assign to jobs for the day. Five jobs are offered with the expected profit in rupees for each machinist on each job being as follows:

| MACHINISTS | JOBS | | | | |
|---|---|---|---|---|---|
| | A | B | C | D | E |
| 1 | 6.20 | 7.80 | 5.00 | 10.10 | 8.20 |
| 2 | 7.10 | 8.40 | 6.10 | 7.30 | 5.90 |
| 3 | 8.70 | 9.20 | 11.10 | 7.10 | 8.10 |
| 4 | 4.80 | 6.40 | 8.70 | 7.70 | 8.00 |

Find the assignment of machinists to jobs that will result in a maximum profit.

8+7