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**Paper Code : MM-401**

# OPERATION RESEARCH AND OPTIMIZATION TECHNIQUES

*Time Allotted* : 3 Hours *Full Marks* : 70

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

## GROUP – A

## ( Multiple Choice Type Questions )

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) The set $S = \{(x_1, x_2) : 0 \leq x_1, x_2 \leq 1\}$ is

      a) a convex set

      b) a concave set

      c) not a convex set

      d) both convex and concave set.

ii) For the system of equations $x_1 + 4x_2 - x_3 = 3$ and $5x_1 + 2x_2 + 3x_3 = 4$,
the value of $x_1 = \frac{5}{9}$, $x_2 = \frac{11}{18}$, $x_3 = 0$ is

a) feasible but not a basic solution

b) not a solution

c) a basic feasible solution

d) a degenerate basic feasible solution.

iii) For a pair of primal and dual problems, if feasible solution exists for both the problems, then

a) both the problems have finite optimal solution

b) finite optimal solution exists for the primal problem only

c) finite optimal solution exists for dual problem only

d) both the problems have unbounded solution.

iv) In a travelling salesman problem, the salesman can visit a city twice, until he has visited all the cities

a) once b) twice

c) thrice d) four times.

v) In a PERT network, the starting vertex is a

a) burst node b) merge node

c) root d) none of these.

vi) A two-person zero-sum game is said to be fair if

a) both the players have equal number of strategies

b) the game has a saddle point

c) the game does not have a saddle point

d) the value of the game is zero.

vii) If in assignment problem, ' k ' be the maximum number of zeroes which can be assigned, then the minimum number of lines which will cover all the zeroes is

a) k b) k + 1

c) k – 1 d) 2k.

viii) A basic solution of the system of equations :

$2x_1 + x_2 - x_3 = 2, 3x_1 + 2x_2 - x_3 = 3$ is

a) (1,1,1) b) (1,1,0)

c) (1,0,0) d) none of these.

ix) A game is solved graphically when the pay-off matrix is of the form

a) $m \times 1$ b) $m \times n$

c) $m \times 2$ d) $n \times m$.

x) If $\lambda$ is the arrival rate, $\mu$ is the service rate, then the average waiting time of a customer (in the queue) is given by

a) $\frac{\lambda}{\lambda(\mu-\lambda)}$ b) $\frac{\lambda}{\lambda(\mu+\lambda)}$

c) $\frac{\lambda}{(\mu-\lambda)}$ d) $\frac{1}{\lambda(\mu-\lambda)}$.

xi) CPM is

a) probabilistic b) deterministic

c) event-oriented d) all of these.

xii) If the $i^{th}$ row of the pay-off matrix of an $m \times n$ rectangular game be dominated by its $r^{th}$ row, then

a) the deletion of the $i^{th}$ row from the matrix does not change the set of optimal strategies of the maximizing player.

b) the deletion of the $r^{th}$ row from the matrix does not change the set of optimal strategies of the maximizing player.

c) both of (a) and (b)

d) none of these.

## GROUP – B

### ( Short Answer Type Questions )

Answer any *three* of the following $3 \times 5 = 15$

2. Solve the following game whose pay-off matrix is given by

Player B

| | | $B_1$ | $B_2$ | $B_3$ |
|---|---|---|---|---|
| Player $A$ | $A_1$ | 1 | 3 | 1 |
| | $A_2$ | 0 | −4 | −3 |
| | $A_3$ | 1 | 5 | −1 |

3. Obtain the dual of the following L.P.P. :

Max $Z = 3x_1 + x_2 + 2x_3 - t$

subject to

$2x_1 - x_2 + 3x_3 + t = 1,$

$x_1 + x_2 - x_3 + t = 3,$

$x_1, x_2 \geq 0$

$x_3, t =$ unrestricted

4. Prove that the number of basic variables in a transportation problem is ( m + n – 1).

5. In { ( M / M / 1 ) : ( ∞ / FIFO) } queue, prove that the average length of the queue is = $\frac{\lambda}{\mu - \lambda}$.

6. Solve the following L.P.P. by simplex method :

*Min* $Z = x_1 + x_2$

subject to

$5x_1 + 9x_2 \leq 45$,

$x_1 + x_2 \geq 2$,

$x_2 \leq 4$, $x_1, x_2 \geq 0$.

## GROUP – C

### ( Long Answer Type Questions )

Answer any *three* of the following. 3 × 15 = 45

7. a) Solve the dynamic programming to solve

Maximize $z = y_1^2 + y_2^2 + y_3^2$

subject to $y_1, y_2, y_3 \leq 4$

and $y_1, y_2, y_3 \geq 0$ and integer. 8

b) Find out the dual of following problem :

Maximize $Z = 3x_1 + x_2 + 2x_3 - x_4$

subject to $2x_1 - x_2 + 3x_3 + x_4 = 1$,

$x_1 + x_2 - x_3 + x_4 = 3$

$x_1 + x_2 \geq 0$ and $x_3$ and $x_4$ are unrestricted in sign.

Show further that the dual of the dual is primal. 7

8. a) A firm makes two types of furniture – chairs and tables. The profit for each product as calculated by the accounting department is Rs. 20 per chair and Rs. 30 per table. Both products are to be processed on three machines $M_1$, $M_2$, $M_3$. The time required in hours by each product and total time available in hours per week on each machine is as follows :

| Machine | Chain | Table | Available Time (hrs) |
|---|---|---|---|
| $M_1$ | 3 | 3 | 36 |
| $M_2$ | 5 | 2 | 50 |
| $M_3$ | 2 | 6 | 60 |

i) Give a mathematical formulation to this linear programming problem.

ii) Use the graphical method to solve this problem. 3 + 4

b) Show that in a pure birth process with mean arrival rate $\lambda$, the probability that there will be $n$ arrivals in time t is $p_n(t) = e^{-\lambda t}\frac{(\lambda t)^n}{n!}$. 8

9. a) The head of the department has five jobs *A*, *B*, *C*, *D*, *E* and five sub-ordinates *V*, *W*, *X*, *Y*, *Z*. The number of hours each sub-ordinate would take to perform each job is as follows :

| | V | W | X | Y | Z |
|---|---|---|---|---|---|
| A | 3 | 5 | 10 | 15 | 8 |
| B | 4 | 7 | 15 | 18 | 8 |
| C | 8 | 12 | 20 | 20 | 12 |
| D | 5 | 5 | 8 | 10 | 6 |
| E | 10 | 10 | 15 | 25 | 10 |

How would the jobs be allocated to minimize the total time ? 7

b) Solve the game using Dominance method whose pay-off matrix is given by

| | | B | | | |
|---|---|---|---|---|---|
| | | B1 | B2 | B3 | B4 |
| A | A1 | 8 | 15 | –4 | –2 |
| | A2 | 19 | 15 | 17 | 16 |
| | A3 | 0 | 20 | 15 | 5 |

8

10. a) Solve the following transportation problem by using VAM method. 6

| | D1 | D2 | D3 | D4 | D5 | Supply |
|---|---|---|---|---|---|---|
| 01 | 73 | 40 | 9 | 79 | 20 | 8 |
| 02 | 62 | 93 | 96 | 8 | 13 | 7 |
| 03 | 96 | 65 | 80 | 50 | 55 | 9 |
| 04 | 57 | 58 | 29 | 12 | 87 | 3 |
| 05 | 56 | 23 | 87 | 18 | 12 | 5 |
| Demand | 6 | 8 | 10 | 4 | 4 | |

b) Use branch and bound method to solve the following LPP :

Maximize $Z = 7x_1 + 9x_2$

subject to $-x_1 + 3x_2 \leq 6;$

$7x_1 + x_2 \leq 35;$

$x_1 \leq 7;$

$x_1, x_2 \geq 0$ and integer. 9

11. a) Provide the optimum job sequencing involving two machines $A$, $B$ of which the processing time (in minutes) is as follows and find the total elapsed time and idle time for each machine. 7

| Machines | Jobs | | | | | | |
|---|---|---|---|---|---|---|---|
| | | $J_1$ | $J_2$ | $J_3$ | $J_4$ | $J_5$ | $J_6$ |
| | **A** | 1 | 3 | 8 | 5 | 6 | 3 |
| | **B** | 5 | 6 | 3 | 2 | 2 | 10 |

b)

| Activity $i-j$ | 1–2 | 1–3 | 1–4 | 2–5 | 3–5 | 4–6 | 5–6 |
|---|---|---|---|---|---|---|---|
| $t_o$ | 1 | 4 | 2 | 2 | 1 | 5 | 6 |
| $t_m$ | 1 | 4 | 2 | 2 | 1 | 5 | 6 |
| $t_p$ | 7 | 7 | 7 | 8 | 1 | 8 | 15 |

where $t_o$ is the optimistic time, $t_p$ is the pessimistic time and $t_m$ is the most likely time.

i) Draw the project network.

ii) Identify all paths through it and write critical path.

iii) Determine the expected project length.

iv) Calculate standard deviation and variance of the project length.

v) What is the percentage of confidence that project will complete–

a) at least 4 weeks earlier than expected time ?

b) no more than 4 weeks later than expected time ? 8

12. a) Establish an E.O.Q model with uniform production, known demand, lead time zero and having shortage which are to be fulfilled. Find the optimum order quantity and optimum cost. 7

b) Solve the following LPP by Big $M$ Method : 8

Maximize $Z = 2x_1 + 9x_2 + x_3$

subject to $x_1 + 4x_2 + 2x_3 \geq 5$,

$3x_1 + x_2 + 2x_3 \geq 4$

and $x_1, x_2, x_3 \geq 0$.