

Name :
Roll No. :
Invigilator's Signature :

**CS/MCA/SEM-4/MM-401/2011
2011
OPERATIONS RESEARCH &
OPTIMIZATION TECHNIQUES**

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

Graph sheet(s) will be supplied by the Institution.

GROUP – A

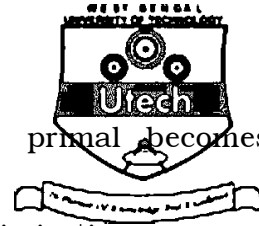
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10
- i) What is the method used to solve an LPP involving artificial variables ?
- a) Dominance method
 - b) Charnes-Big M method
 - c) VAM
 - d) None of these.
- ii) The optimality condition for minimization LPP in the simplex method is
- a) $z_j - c_j \geq 0, \forall_j$
 - b) $z_j - c_j > 0, \forall_j$
 - c) $z_j - c_j < 0, \forall_j$
 - d) $z_j - c_j \leq 0, \forall_j$.

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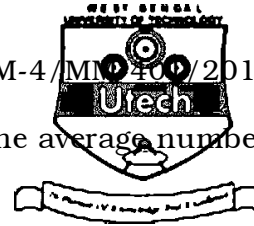
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- iii) The maximization problem in the primal becomes problem in its dual.
 - a) minimization
 - b) maximization
 - c) max-min
 - d) min-max.
- iv) For a travelling salesman problem who has visited n cities, the number of possible routes are
 - a) $n!$
 - b) $(n + 1)!$
 - c) $(n - 1)!$
 - d) $n - 1$.
- v) The basic feasible solution of the system of equations $x_1 + x_2 + x_3 = 8, 3x_1 + 2x_2 = 18$ are
 - a) no basic solution
 - b) $(2, 6, 0), (6, 0, 2)$
 - c) $(1, 7, 0), (7, 1, 0)$
 - d) none of these.
- vi) In an assignment problem, the minimum number of lines covering all zeros in the reduced cost matrix of order n can be
 - a) at least n
 - b) $n + 1$
 - c) $n - 1$
 - d) at most n .
- vii) Which of the following is not a method to obtain the basic feasible solution in transportation problem ?
 - a) VAM
 - b) Least cost method
 - c) North-West corner method
 - d) MODI method.
- viii) A two-person zero-sum game is said to be fair if
 - a) both the players have equal number of strategies
 - b) the game has a saddle point
 - c) the game does not have a saddle point
 - d) the value of the game is zero.

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ix) In an $(M/M/1) : (\infty/FIFO)$ model, the average number of customers $E (n) =$

- a) ρ^n
- b) $\frac{\rho}{1 + \rho}$
- c) $\frac{\rho^2}{1 + \rho}$
- d) none of these.

Here the symbols have their usual meanings.

x) Number of basic variables in a balanced transportation problem is

- a) mn
- b) $m + n - 1$
- c) $m + n$
- d) $(m - 1) (n - 1)$.

xi) In PERT analysis, the variance of a job having optimistic time 5, pessimistic time 17 and most likely time 8, is

- a) 3
- b) 4
- c) 7
- d) none of these.

xii) CPM is

- a) probabilistic
- b) deterministic
- c) event oriented
- d) all of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

- 2. Prove that the vectors $(1, 1, 1)$, $(1, 1, 0)$ and $(1, 0, 0)$ form a basis in E^3 . Prove also that the vector $(1, 3, 1)$ can replace any of three vectors of the basis to form a new basis.
- 3. Prove that the number of basic variables in a transportation problem is $(m + n - 1)$.

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4. If the arrival rate is λ and service rate is μ , then prove that the expected queue length is $\frac{\lambda^2}{\mu(\mu - \lambda)}$.

5. Find any two basic feasible solutions of the following set of equations.

$$2x_1 + 3x_2 - x_3 + 4x_4 = 8$$

$$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$$

$$x_1, x_2, x_3, x_4 \geq 0.$$

6. Solve the game whose pay-off matrix is given by

$$\begin{pmatrix} -5 & 3 & 1 & 20 \\ 5 & 5 & 4 & 6 \\ -4 & -2 & 0 & -5 \end{pmatrix}$$

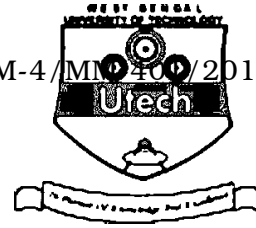
GROUP - C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) A firm manufactures products A and B and sells them at a profit of Rs. 2 and Rs. 3 respectively. Each product is processed on two machines I and II. Type A requires 1 minute processing on machine I and 2 minutes in machine II. Type B takes 1 minute in both machines. Machine I is available for not more than 6 hours and 40 minutes, while machine II is available not more than 10 hours in any working day. Formulate this LPP mathematically and solve by graphical method.

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b) Find the dual of the following LPP :

Minimize $Z = x_1 - x_2$

Subject to $2x_1 + x_2 \geq 2$

$-x_1 - x_2 \geq 1, x_1, x_2 \geq 0.$ 12 + 3

8. a) Solve the following LPP :

Maximize $Z = x_1 + x_2 + x_3$

Subject to $3x_1 + 2x_2 + x_3 \leq 3$

$2x_1 + x_2 + 2x_3 \leq 2$

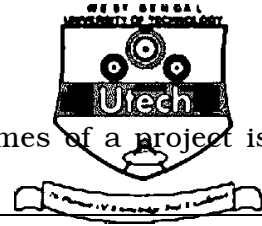
and $x_1, x_2, x_3 \geq 0.$

b) Solve the following transportation problem by VAM and find out the optimal solution :

	D_1	D_2	D_3	D_4	a_i
O_1	3	8	7	4	30
O_2	5	2	9	5	50
O_3	4	3	6	2	80
b_i	20	60	55	40	

7 + 8

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9. Optimistic, most likely and pessimistic times of a project is given below :

Activity	Estimated Duration (in weeks)		
	Optimistic	Most likely	Pessimistic
1 — 2	1	1	7
1 — 3	1	4	7
1 — 4	2	2	8
2 — 5	1	1	1
3 — 5	2	5	14
4 — 6	2	5	8
5 — 6	3	6	15

- a) Draw the project network.
- b) Find the expected duration and variance of each activity.
- c) Calculate the early and late occurrence for each event and the expected project length.
- d) Calculate the variance and standard deviations of project length.

What is the probability that the project will be completed

- i) 4 weeks earlier than expected ?
- ii) not more than 4 weeks later than expected ?
- e) If the project due date is 19 weeks, what is the probability of meeting the due date ?

[Given : $\Phi (1.33) = 0.4082$ and $\Phi (0.666) = 0.2514$]

$$4 + 2 + 2 + (2 + 2) + 3$$

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10. a) Find the dual of the following LPP and hence solve it :

Maximize $Z = 3x_1 - 2x_2$

Subject to $x_1 \leq 4$

$$x_2 \leq 6$$

$$x_1 + x_2 \leq 5$$

$$-x_2 \leq -1, x_1, x_2 \geq 0.$$

b) Show that in a pure birth process with mean arrival rate λ , the probability that there will be n arrivals in time t is

$$P_n(t) = e^{-\lambda t} (\lambda t)^n / n! \quad 7 + 8$$

11. a) Use dynamic programming to solve the problem :

Minimize $Z = y_1^2 + y_2^2 + y_3^2$

Subject to $y_1 + y_2 + y_3 \geq 15$ and $y_1, y_2, y_3 \geq 0$.

b) Establish EOQ model with uniform production rate, known demand, lead time zero and not shortage. Find the optimum order quantity and the optimum cost.

8 + 7

