Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/MCA/SEM-4/MM-401/2010

2010

OPERATION RESEARCH AND OPTIMIZATION TECHNIQUES

Time Allotted : 3 Hours *Full Marks* : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Graph sheet(s) will be provided by the Institution.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) What is the method used to solve an LPP involving artificial variables ?

      a) Dominance method

      b) Charnes-Big M method

      c) VAM

      d) None of these.

   ii) The optimality condition for minimization LPP in the simplex method is

      a) $Z_j - C_j \geq 0 \ \forall j$ b) $Z_j - C_j > 0 \ \forall j$

      c) $Z_j - C_j < 0 \ \forall j$ d) $Z_j - C_j \leq 0 \ \forall j$.

iii) The maximization problem in the primal becomes ..... problem in its dual.

a) minimization b) maximization

c) max-min d) min-max.

iv) For a travelling salesman problem who has visited n cities, the number of possible routes are

a) n ! b) $(n + 1)$!

c) $(n - 1)$! d) $n - 1$.

v) In an assigment problem, the minimum number of lines covering all zeros in the reduced cost matrix of order n can be

a) at least n b) $n + 1$

c) $n - 1$ d) at most n.

vi) The basic feasible solutions of the system of equations

$x_1 + x_2 + x_3 = 8,$

$3x_1 + 2x_2 = 18$ are

a) no basic solution

b) (2, 6, 0), (6, 0, 2)

c) (1, 7, 0), (7, 1, 0)

d) none of these.

vii) Which of the following is not a method to obtain the initial basic feasible solution in transportation problem ?

a) VAM

b) Least cost method

c) North-West corner method

d) MODI Method.

viii) A two-person zero-sum game is said to be fair if

a) both the players have equal number of strategies

b) the game has a saddle point

c) the game does not have a saddle point

d) the value of the game is zero.

ix) In networking problem, the activity for which total float is 0, is called

a) critical activity

b) independent ativity

c) probabilistic activity

d) none of these.

x) In a game theory problem, saddle point occurs when

a) max (row min) = min (column maximum)

b) min (row min) = min (column maximum)

c) max (row min) = max (column maximum)

d) none of these.

xi) In a queue, if (λ/μ) is greater than 1, then the

a) system is transient

b) system is in steady state

c) system is in explosive state

d) none of these.

xii) In the case of degeneracy while solving transportation problem, the small allocation is made in

a) non-occupied cell

b) occupied cell

c) a non-occupied cell in independent position

d) none of these.

xiii) In a purchasing model without shortage, if C1 is the holding cost per unit per time, C3 is the set-up cost per order and R is the demand rate, then EOQ is equal to

a) $(2C_3 R / C_1)^{1/2}$ b) $(2C_1 R / C_3)^{1/2}$

c) $(2C_1 C_3 R)^{1/2}$ d) none of these.

xiv) In PERT analysis, the variance of a job having optimistic time 5, pessimistic time 17, and most likely time 8, is

a) 3 b) 4

c) 7 d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. A company makes two kinds of leather belts *A* and *B*. Their respective unit profits are Rs. 4 and Rs. 3. One belt of type *A* requires 2 hours and type *B* requires 1 hour of time in making. The total man-hours available are 1000 per day. Due to insufficient supply of leather, the company can make only 800 belts per day. Only 400 buckles for type *A* and 700 buckles for type *B* are available. Formulate the problem as an L.P.P. and solve it graphically.

3. Find out the dual of the problem

Maximize, $Z = 2x_1 + 3x_2 + 4x_3$

subject to $3x_1 + x_2 + x_3 \leq 2$

$-4x_1 + 3x_3 \geq 4$

$x_1 - 5x_2 + x_3 = 5$

$x_1 \geq 0, x_2 \geq 0$ and x_3 is unrestricted in sign.

4. Solve the following game graphically :

$$\text{Player } A \begin{bmatrix} 3 & -3 & 4 \\ -1 & 1 & -3 \end{bmatrix} \quad \text{Player } B$$

5. Solve the following D.P.P :

Minimize $Z = y_1^2 + y_2^2 + y_3^2$

subject to $y_1 + y_2 + y_3 \geq 15$

$y_1, y_2, y_3 \geq 0.$

6. What is Economic Order Quantity (EOQ) ? Derive an Economic Order Quantity (EOQ) model with uniform rate of demand, infinite production rate and having no shortage.

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Use simplex method to solve the following problem :

Maximize, $Z = 2x_1 + 5x_2$

subject to $x_1 + 4x_2 \leq 24$

$3x_1 + x_2 \leq 21,$

$x_1 + x_2 \leq 9$

$x_1 \geq 0, x_2 \geq 0.$

b) Use big M method to maximize

$Z = 6x_1 + 4x_2$

subject to the constraints :

$2x_1 + 3x_2 \leq 30$

$3x_1 + 2x_2 \leq 24$

$x_1 + x_2 \geq 3$

$x_1 \geq 0, x_2 \geq 0.$ 8 + 7

8. a) Use Dijkstra's algorithm to determine the shortest path and length of the shortest path from A to E for the following network :

Dia.

b) Using dynamic programming determine the value of u_1, u_2, u_3 so as to maximize

$Z = u_1 . u_2 . u_3$, subject to the constraints

$u_1 + u_2 + u_3 = 10,$

$u_1, u_2, u_3 \geq 0.$ 10 + 5

9. A small maintenance project cosists of the following jobs whose precedence relationship is given below :

	Estimated Duration (weeks)		
Activity	***Optimistic***	***Most Likely***	***Pessimistic***
1-2	1	1	7
1-3	1	4	7
1-4	2	2	8
2-5	1	1	1
3-5	2	5	14
4-6	2	5	8
5-6	3	6	15

a) Draw the project network.

b) Find the expected duration and variance of each activity.

c) Calculate the early and late occurrence for each event and the expected project length.

d) Calculate the variance and standard deviations of project length. What is the probability that the project will be completed :

i) 4 weeks earlier than expected ?

ii) not more than 4 weeks later than expected ?

e) If the project due date is 19 weeks, what is the probability of meeting the due data ?

[Given that ϕ (1·33) = 0·4082 and

Φ (0·666) = 0·2514] . 4 + 2 + 2 + (2 + 2) + 3

10. a) Find the dual of the following LPP and hence solve it :

Maximize $Z = 3x_1 - 2x_2$

subject to $x_1 \leq 4$

$x_2 \leq 6$

$x_1 + x_2 \leq 5$

$-x_2 \leq -1 \qquad x_1, x_2 \geq 0$

b) Customers arrive at a one-window drive in bank according to Poisson distribution with 10 cars per hour. Service time per customer is exponential with mean 6 minutes. The space in front of the window including that for the serviced car can accommodate a maximum of 3 cars. Other can wait outside its space.

i) What is the probability that an arriving customer can drive directly to the window ?

ii) What is probability that an arriving customer will have to wait outside the indicated space ?

iii) How long is the arriving customer expected to wait before starting service ? 8 + 7

11. Find the optimal solution to the following integer programming problem :

Maximize $Z = x_1 - x_2$

subject to $x_1 + 2x_2 \leq 4$

$6x_1 + 2x_2 \leq 9$

$x_1, x_2 \geq 0$ and x_1, x_2 are integers.