



Name :

Roll No. :

Invigilator's Signature :

**CS/MCA/SEM-4/MM-401/2010
2010**

**OPERATION RESEARCH AND OPTIMIZATION
TECHNIQUES**

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

Graph sheet(s) will be provided by the Institution.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following :

10 × 1 = 10

- i) What is the method used to solve an LPP involving artificial variables ?
- a) Dominance method
 - b) Charnes-Big M method
 - c) VAM
 - d) None of these.
- ii) The optimality condition for minimization LPP in the simplex method is
- a) $Z_j - C_j \geq 0 \forall j$
 - b) $Z_j - C_j > 0 \forall j$
 - c) $Z_j - C_j < 0 \forall j$
 - d) $Z_j - C_j \leq 0 \forall j$.



iii) The maximization problem in the primal becomes problem in its dual.

- a) minimization b) maximization
- c) max-min d) min-max.

iv) For a travelling salesman problem who has visited n cities, the number of possible routes are

- a) $n!$ b) $(n + 1)!$
- c) $(n - 1)!$ d) $n - 1$.

v) In an assignment problem, the minimum number of lines covering all zeros in the reduced cost matrix of order n can be

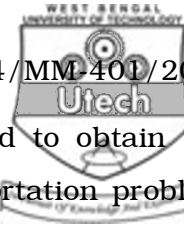
- a) at least n b) $n + 1$
- c) $n - 1$ d) at most n .

vi) The basic feasible solutions of the system of equations

$$x_1 + x_2 + x_3 = 8,$$

$$3x_1 + 2x_2 = 18 \text{ are}$$

- a) no basic solution
- b) $(2, 6, 0), (6, 0, 2)$
- c) $(1, 7, 0), (7, 1, 0)$
- d) none of these.



- vii) Which of the following is not a method to obtain the initial basic feasible solution in transportation problem ?
- a) VAM
 - b) Least cost method
 - c) North-West corner method
 - d) MODI Method.
- viii) A two-person zero-sum game is said to be fair if
- a) both the players have equal number of strategies
 - b) the game has a saddle point
 - c) the game does not have a saddle point
 - d) the value of the game is zero.
- ix) In networking problem, the activity for which total float is 0, is called
- a) critical activity
 - b) independent activity
 - c) probabilistic activity
 - d) none of these.
- x) In a game theory problem, saddle point occurs when
- a) $\max(\text{row min}) = \min(\text{column maximum})$
 - b) $\min(\text{row min}) = \min(\text{column maximum})$
 - c) $\max(\text{row min}) = \max(\text{column maximum})$



- d) none of these.
- xi) In a queue, if (λ/μ) is greater than 1, then the
- a) system is transient
 - b) system is in steady state
 - c) system is in explosive state
 - d) none of these.
- xii) In the case of degeneracy while solving transportation problem, the small allocation is made in
- a) non-occupied cell
 - b) occupied cell
 - c) a non-occupied cell in independent position
 - d) none of these.
- xiii) In a purchasing model without shortage, if C_1 is the holding cost per unit per time, C_3 is the set-up cost per order and R is the demand rate, then EOQ is equal to
- a) $(2C_3 R / C_1)^{1/2}$
 - b) $(2C_1 R / C_3)^{1/2}$
 - c) $(2C_1 C_3 R)^{1/2}$
 - d) none of these.
- xiv) In PERT analysis, the variance of a job having optimistic time 5, pessimistic time 17, and most likely time 8, is
- a) 3
 - b) 4
 - c) 7
 - d) none of these.



GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. 3 × 5 = 15

2. A company makes two kinds of leather belts A and B. Their respective unit profits are Rs. 4 and Rs. 3. One belt of type A requires 2 hours and type B requires 1 hour of time in making. The total man-hours available are 1000 per day. Due to insufficient supply of leather, the company can make only 800 belts per day. Only 400 buckles for type A and 700 buckles for type B are available. Formulate the problem as an L.P.P. and solve it graphically.

3. Find out the dual of the problem

$$\begin{aligned} &\text{Maximize, } Z = 2x_1 + 3x_2 + 4x_3 \\ &\text{subject to } 3x_1 + x_2 + x_3 \leq 2 \\ &\quad \quad \quad -4x_1 + 3x_3 \geq 4 \\ &\quad \quad \quad x_1 - 5x_2 + x_3 = 5 \\ &\quad \quad \quad x_1 \geq 0, x_2 \geq 0 \text{ and } x_3 \text{ is unrestricted in sign.} \end{aligned}$$

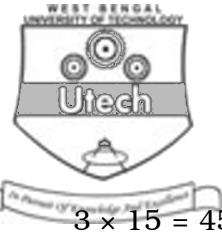
4. Solve the following game graphically :

$$\begin{array}{c} \text{Player B} \\ \text{Player A} \begin{bmatrix} 3 & -3 & 4 \\ -1 & 1 & -3 \end{bmatrix} \end{array}$$

5. Solve the following D.P.P :

$$\begin{aligned} &\text{Minimize } Z = y_1^2 + y_2^2 + y_3^2 \\ &\text{subject to } y_1 + y_2 + y_3 \geq 15 \\ &\quad \quad \quad y_1, y_2, y_3 \geq 0. \end{aligned}$$

6. What is Economic Order Quantity (EOQ) ? Derive an Economic Order Quantity (EOQ) model with uniform rate of demand, infinite production rate and having no shortage.



GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following.

3 × 15 = 45

7. a) Use simplex method to solve the following problem :

$$\text{Maximize, } Z = 2x_1 + 5x_2$$

$$\text{subject to } x_1 + 4x_2 \leq 24$$

$$3x_1 + x_2 \leq 21,$$

$$x_1 + x_2 \leq 9$$

$$x_1 \geq 0, x_2 \geq 0.$$

- b) Use big M method to maximize

$$Z = 6x_1 + 4x_2$$

subject to the constraints :

$$2x_1 + 3x_2 \leq 30$$

$$3x_1 + 2x_2 \leq 24$$

$$x_1 + x_2 \geq 3$$

$$x_1 \geq 0, x_2 \geq 0.$$

8 + 7

8. a) Use Dijkstra's algorithm to determine the shortest path and length of the shortest path from A to E for the following network :

Dia.



- b) Using dynamic programming determine the value of u_1, u_2, u_3 so as to maximize

$$Z = u_1 \cdot u_2 \cdot u_3, \text{ subject to the constraints}$$

$$u_1 + u_2 + u_3 = 10,$$

$$u_1, u_2, u_3 \geq 0.$$

10 + 5

9. A small maintenance project consists of the following jobs whose precedence relationship is given below :

| Activity | Estimated Duration (weeks) | | |
|----------|------------------------------|-------------|-------------|
| | Optimistic | Most Likely | Pessimistic |
| 1-2 | 1 | 1 | 7 |
| 1-3 | 1 | 4 | 7 |
| 1-4 | 2 | 2 | 8 |
| 2-5 | 1 | 1 | 1 |
| 3-5 | 2 | 5 | 14 |
| 4-6 | 2 | 5 | 8 |
| 5-6 | 3 | 6 | 15 |

- Draw the project network.
- Find the expected duration and variance of each activity.
- Calculate the early and late occurrence for each event and the expected project length.
- Calculate the variance and standard deviations of project length. What is the probability that the project will be completed :
 - 4 weeks earlier than expected ?
 - not more than 4 weeks later than expected ?
- If the project due date is 19 weeks, what is the probability of meeting the due data ?

[Given that $\phi (1.33) = 0.4082$ and

$\Phi (0.666) = 0.2514$].

$4 + 2 + 2 + (2 + 2) + 3$



10. a) Find the dual of the following LPP and hence solve it :

$$\begin{aligned} \text{Maximize} \quad & Z = 3x_1 - 2x_2 \\ \text{subject to} \quad & x_1 \leq 4 \\ & x_2 \leq 6 \\ & x_1 + x_2 \leq 5 \\ & -x_2 \leq -1 \quad x_1, x_2 \geq 0 \end{aligned}$$

b) Customers arrive at a one-window drive in bank according to Poisson distribution with 10 cars per hour. Service time per customer is exponential with mean 6 minutes. The space in front of the window including that for the serviced car can accommodate a maximum of 3 cars. Other can wait outside its space.

- i) What is the probability that an arriving customer can drive directly to the window ?
- ii) What is probability that an arriving customer will have to wait outside the indicated space ?
- iii) How long is the arriving customer expected to wait before starting service ? 8 + 7

11. Find the optimal solution to the following integer programming problem :

$$\begin{aligned} \text{Maximize} \quad & Z = x_1 - x_2 \\ \text{subject to} \quad & x_1 + 2x_2 \leq 4 \\ & 6x_1 + 2x_2 \leq 9 \\ & x_1, x_2 \geq 0 \text{ and } x_1, x_2 \text{ are integers.} \end{aligned}$$
