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**ENGINEERING & MANAGEMENT EXAMINATIONS, JUNE - 2009**  
**OPERATIONS RESEARCH AND OPTIMIZATION TECHNIQUES**  
**SEMESTER - 4**



Time : 3 Hours ]

[ Full Marks : 70

Graph sheets are provided at the end of the booklet.

**GROUP - A****( Multiple Choice Type Questions )**

1. Choose the correct alternatives for any *ten* of the following : 10 × 1 = 10
- i) Given a system of  $m$  simultaneous equations in  $n$  unknowns ( $m < n$ ), the number of basic variables will be
- a)  $m$  b)  $n$   
 c)  $m - n$  d)  $m + n$  .
- ii) A two-person zero-sum game is said to be fair if
- a) both the players have equal number of strategies  
 b) the game has a saddle point  
 c) the game does not have a saddle point  
 d) the value of the game is zero.
- iii) In an assignment problem involving four workers and three jobs, the total number of assignments possible is
- a) 4 b) 3  
 c) 7 d) 21.
- iv) In  $\{ ( M/M/1 ) : ( \infty /F/FO ) \}$ , average length of a non-empty queue is
- a)  $\frac{\lambda^2}{\mu (\mu - \lambda)}$  b)  $\frac{\mu}{\mu - \lambda}$   
 c)  $\frac{\lambda\mu}{(\mu - \lambda)^2}$  d) none of these.
- v) In a flow pattern  $\sum f_{tA} = \sum f_{At}$  when the vertex  $A$  is
- a) arbitrary vertex b) any vertex other than source  
 c) source d) none of these.



vi) The formula for finding the minimum inventory cost under the purchasing model without shortage is

a)  $\sqrt{2RC_1C_3}$

b)  $\frac{\sqrt{2C_3R}}{C_1}$

c)  $\frac{\sqrt{C_1}}{2RC_3}$

d) none of these.




vii) A simplex in two dimension is

a) rectangle

b) line segment

c) triangle

d) pentagon.

viii) When a positive quantity  $R$  is divided into five parts, the maximum value of their product is

a)  $5k$

b)  $(k/5)^5$

c)  $(k5)^5$

d)  $5(k/5)$ .

ix) The optimality condition for minimization LPP in the simplex method is

a)  $Z_j - C_j \geq 0$

b)  $Z_j - C_j \leq 0$

c)  $Z_j - C_j < 0$

d) none of these.

x) What is the method used to solve an LPP involving artificial variables ?

a) Simplex method

b) Charnes  $M$  method

c) VAM

d) None of these.

xi) Consider the following game :

Player (A)


Player (B)		1	2	3
1		5	50	50
2		1	1	0.1
3		10	1	10

Then the value of the game is

a)  $\frac{10}{3}$

b)  $\frac{50}{7}$



- c)  $\frac{55}{6}$  d) 50. 
- xii) The total number of possible solutions for  $n \times n$  assignment problem is always
- a)  $n$  b)  $n - 1$   
 c) 1 d)  $n!$
- xiii) The point of intersection of pure strategies in a game is called
- a) value of the game b) saddle point  
 c) mixed strategy d) optimal strategy.
- xiv) The system of simultaneous equations given by
- $$2x_1 + 3x_2 + 4 = 0$$
- $$3x_1 + 4x_2 + 6 = 0$$
- $$4x_1 + 5x_2 + 8 = 0$$
- is
- a) consistent b) inconsistent  
 c) possessing unique solution d) none of these.

### GROUP – B

#### ( Short Answer Type Questions )

Answer any *three* of the following.

3 × 5 = 15

2. Solve the following LPP by graphical method :

Minimize  $Z = 20x_1 + 10x_2$

Subject to  $x_1 + 2x_2 \leq 40$

$$3x_1 + x_2 \geq 30$$

$$4x_1 + 3x_2 \geq 60$$



$$x_1, x_2 \geq 0$$



3. Arrivals at a telephone booth are considered to be Poisson with an average time of 10 minutes between one arrival and the next. The length of the phone call is assumed to be distributed exponentially with mean 3 minutes.

- i) What is the probability that a person arriving at the booth will have to wait ?
- ii) The telephone department will install a second booth when convinced that an arrival would expect waiting for at least 3 minutes for a phone call. By how much should the flow of arrivals increase in order to justify a second booth ?
- iii) What is the average length of the queue that forms time to time ?

4. Find the optimal strategies and the value of the game  $G$  whose pay-off matrix is

	<i>Player B</i>	
<i>Player A</i>	- 2	6
	5	1

5. There are five jobs, each of which is to be processed through two machines  $M_1$  and  $M_2$  in the order  $M_1 M_2$ . The processing hours are the following :

<i>Jobs</i>	1	2	3	4	5
$M_1$	3	8	5	7	4
$M_2$	4	10	6	5	8

Determine the optimal sequence of the five jobs, the minimum elapsed time and the ideal times for the machines  $M_1$  and  $M_2$ .

6. Find the basic solution or solutions, if there be any, of the set of equations

$$2x_1 + 4x_2 - 2x_3 = 1$$

$$10x_1 + 3x_2 - 7x_3 = 33$$

7. Solve the assignment problem :

	1	2	3
<i>A</i>	7	5	6
<i>B</i>	8	4	7



C	9	6	4
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**GROUP - C****( Long Answer Type Questions )**Answer any *three* questions.

3 × 15 = 45

8. a) Solve the following LPP by Simplex method. 7
- Maximize  $Z = 60x_1 + 50x_2$
- subject to
- $x_1 + 2x_2 \leq 40$
- $3x_1 + 2x_2 \leq 60$
- $x_1, x_2 \geq 0$
- b) Solve the following linear programming problem by Charnes Big M method ( if possible ) :
- Maximize  $Z = 2x_1 - x_2 + 5x_3$
- subject to
- $x_1 + 2x_2 + 2x_3 \leq 2$
- $\frac{5}{2}x_1 + 3x_2 + 4x_3 = 12$
- $4x_1 + 3x_2 + 2x_3 \geq 24$
- and  $x_1, x_2, x_3 \geq 0$
- What is the optimal value of  $Z$  ? 8
9. a) Write down the dual of the following LPP :
- Minimize  $Z = 3x_1 + x_2$
- subject to
- $2x_1 + x_2 \geq 14,$
- $x_1 - x_2 \geq 4, x_1, x_2 \geq 0,$
- and solving the dual problem find out the optimal solution and the optimal value of the objective function. 8
- b) Find the optimal solution to the following integer programming problem : 7
- Maximize  $Z = x_1 - x_2$



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subject to  $x_1 + 2x_2 \leq 4$ ,

$6x_1 + 2x_2 \leq 9, x_1, x_2 \geq 0$  and

$x_1, x_2$  are integers.



10. a) Solve the following balanced Transportation Problem :

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	$D_1$	$D_2$	$D_3$	$D_4$	Capacity
F1	2	3	11	7	6
F2	1	0	6	1	1
F3	5	8	15	9	10
Requirement	7	5	3	2	17

b) Use dynamic programming to solve the following problems :

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$$\text{Minimize } Z = y_1^2 + y_2^2 + y_3^3$$

subject to  $y_1 + y_2 + y_3 \geq 15$

and  $y_1, y_2, y_3 \geq 0$ .

11. a) Solve the following two-person-zero game using the method of dominance :

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		Player B			
Player A	5	-10	9	0	
	6	7	8	1	
	8	7	15	1	
	3	4	-1	4	

b) Find the optimal assignments to find the minimum cost for the assignment following cost matrix :

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	J1	J2	J3
P1	12	24	15
P2	23	18	24
P3	30	14	28

12. Use dynamic programming to solve

$$\text{Maximum } Z = Y_1, Y_2, Y_3$$





Subject to constraints

$$Y_1 + Y_2 + Y_3 = 5$$

$$\text{and } Y_1, Y_2, Y_3 \geq 0.$$



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13. a) Write the differences between PERT and CPM.

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b) The following information is given :

Activity	1 - 2	2 - 3	2 - 4	3 - 5	4 - 6	5 - 6	5 - 7	6 - 7
Pessimistic time ( weeks )	3	9	6	8	8	0	5	8
Most likely time ( weeks )	3	6	4	6	6	0	4	5
Optimistic time ( weeks )	3	3	2	4	4	0	3	3

Draw the network diagram for the above. Calculate

- i) Variance of each activity
- ii) Critical path and expected project length
- iii) The probability that the project will be completed in 23 weeks.

Given :

Z-value	1.9	1.91	1.92	1.93	1.94
Probability	0.9713	0.9719	0.9726	0.9732	0.9738

$$4 + 2 + ( 2 + 1 ) + 4$$

14. a) Assuming that the expected times are normally distributed, find the probability of meeting the schedule date for the given network :

Job	1 - 2	1 - 3	2 - 4	3 - 4	4 - 5	3 - 5
$t_o$	2	9	5	2	6	8



$t_m$	5	12	14	5	6	17
$t_p$	14	15	17	12	12	20

Schedule project completion date is 30 days. Also find the date on which the project manager can complete the project with a probability of 0.90. 8

- b) A small project consists of seven activities for which the relevant data are given :

Activity	Immediate predecessors	Time ( days )
A	-	2
B	-	1
C	A	3
D	A, B	2
E	C, D	1
F	B, D	3
G	E, F	1

- i) Draw the network diagram.  
 ii) Indicate the critical path and calculate the total float and free float for each activity. 7

15. a) The following network gives the distance in miles between pairs of entries 1, 2, ..... and 8. Find the shortest rout-between city 1 and 8 using Dijkstras Algorithm. 8

Dia.

- b) The following table shows the jobs of a network along with their time estimates. The time estimates are in days :

Job	1 - 2	1 - 6	2 - 9	2 - 4	3 - 5	4 - 5	5 - 8	6 - 7	7 - 8
$t_o$	3	2	6	2	5	3	1	3	4
$t_m$	6	5	12	5	11	6	4	9	9



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$t_b$	15	14	30	8	17	15	7	27	28
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Draw the project network.

- i) Find the critical path.
- ii) Find the probability that the project is completed in 31 days.  
 $[ P ( z \leq - 2.1667 ) = 0.0114 ]$ .



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END