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CS/MCA/SEM-4/MM-401/2013

2013
OPERATIONS RESEARCH AND OPTIMIZATION TECHNIQUES

Time Allotted : 3 Hours *Full Marks :* 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Graph sheet(s) will be provided by the Institute on demand.

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

   i) If a variable x_i is unrestricted in sign in a primal L.P.P., then the corresponding dual *i*th constraint in the dual problem will be

      a) ≥ type b) ≤ type

      c) equality constraint d) none of these.

   ii) In a (M/M/1 : ∞ / FIFO) queue model with arrival and service rates λ and μ (λ < μ) respectively, the 'Average length of the non-empty queue' [$E(m / m > 0)$] is given by

      a) λ / (μ – λ) b) μ / (μ – λ)

      c) 1 / (μ – λ) d) 1 / (λ – μ).

iii) If some constant is added to each cost C_{ij} of the assignment matrix then the

a) optimal solution changes

b) optimal solution remains same

c) optimal solution does not exist

d) none of these.

iv) For two-person game with A and B, the minimizing and maximizing players respectively, the optimum strategies are

a) Minimax for A and Maximin for B

b) Maximax for A and Minimax for B

c) Minimin for A and Maximin for B

d) Maximin for A and Minimax for B.

v) Which one of the following is not a deterministic method ?

a) L.P.P b) T.P

c) C.P.M d) P.E.R.T.

vi) Let the time estimates for a particular activity be t_0 = 5 days, t_m = 7 days, t_p = 9 days. Then the expected time t_c is (where the symbols have their usual meanings)

a) 10 days b) 15 days

c) 5 days d) 7 days.

vii) The value of the game having the following pay-off matrix is

	B_1	B_2	B_3
A_1	10	2	3
A_2	7	6	8
A_3	0	3	1

a) 6 b) 10

c) 3 d) 2.

viii) In a simple deterministic EOQ model, with constant demand rate (D) and infinite rate of production, the economic lot size is

a) $\sqrt{2K/Dh}$ b) $\sqrt{2/KDh}$

c) $\sqrt{2KDh}$ d) $\sqrt{2KD/h}$.

ix) In an assignment problem, the minimum number of lines covering all the zeros in the reduced cost matrix of order n can be

a) at most n b) $n + 1$

c) $n - 1$ d) at least n.

x) Consider the unit simplex $S = \{ (x, y, z) : x + y + z \leq 1, x, y, z \geq 0 \}$. Then the number of vertices S has

a) 2 b) 4

c) 5 d) 7.

xi) To find critical path, in the forward pass calculations, we calculate for every tail node of each activity in the

a) latest start time b) latest finish time

c) earliest start time d) earliest finish time.

xii) To find critical path, in the backward pass calculations, we calculate for every head node of each activity in the

a) latest finish time b) total float

c) free float d) earliest start time.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Solve the following LPP by graphical method and then find the optimal solution, if exists :

Minimize $Z = x_1 + x_2$

Subject to $5x_1 + 9x_2 \leq 45$

$x_1 + x_2 \geq 2$

$x_2 \leq 4$

and $x_1, x_2 \geq 0.$

3. Find the dual of the following LPP :

Maximize $Z = 2x_1 + 7x_2 + 5x_3$

Subject to $2x_1 + 5x_2 + 7x_3 \leq 17$

$3x_1 + 2x_2 + 5x_3 = 13$

$5x_1 + 3x_2 + x_3 \leq 9$

and $x_1, x_3 \geq 0$, x_2 unrestricted in sign.

4. Solve the following LPP by simplex algorithm :

Maximize $Z = 2x_1 + x_2$

Subject to $2x_1 + 5x_2 \leq 17$

$3x_1 + 2x_2 \leq 10$

and $x_1, x_2 \geq 0.$

5. Find out the initial basic feasible solution and corresponding transportation cost of the following transportation problem by using VAM.

	D	*E*	*F*	*G*	Available
A	16	18	22	19	150
B	21	23	19	15	200
C	26	29	18	15	300
Demand	125	200	175	150	

6. Use dominance property to reduce the payoff matrix and solve the game.

0	0	0	0	0	0
4	2	0	2	1	1
4	3	1	3	2	2
4	3	7	– 5	1	2
4	3	4	– 1	2	2
4	3	3	– 2	2	2

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) Find out the dual of the following problem :

Maximize $Z = 2x_1 + 3x_2 - 4x_3$

Subject to $3x_1 + x_2 + x_3 \leq 2$

$-4x_1 + 3x_3 \geq 4$

$x_1 - 5x_2 + x_3 = 5$

$x_1, x_2 \geq 0$ and x_3 is unrestricted. 7

b) Solve the dynamic programming to solve

Minimize $Z = y_1^2 + y_2^2 + y_3^2$

Subject to $y_1 + y_2 + y_3 \geq 15$

and $y_1, y_2, y_3 \geq 0$. 8

8. a) Arrival rate of telephone calls of a telephone booth is according to Poisson distribution with an average time of 9 minutes between two successive arrivals. The length of telephone call is assumed to be exponential distribution with mean 3 minutes.

i) Find the time that a person arriving at the booth will have to wait

ii) Find average queue length

iii) The telephone company will install a second booth when they been convinced that an arrival would expected to have to wait at least 4 minutes for the phone.

Find the increase in flow of arrivals which will justify a second booth.

b) A company uses annual 50000 units of an item, each costing Rs. 1·20. Each order costs Rs. 45 and inventory carrying costs are 15% of annual average inventory value.

i) Find EOQ.

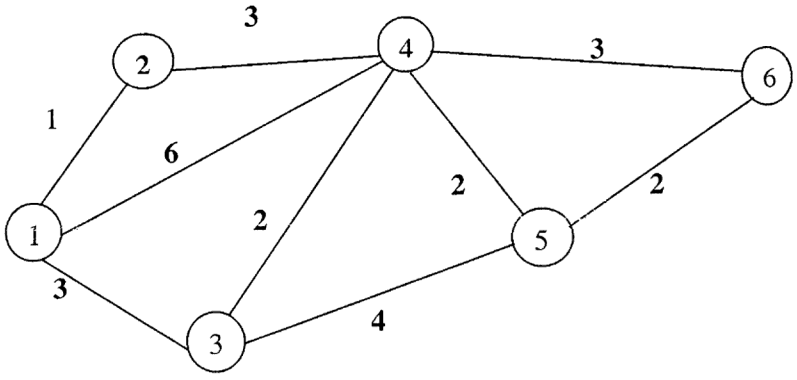
ii) If the company operates 250 day a year, the procurement time is 10 day and safety stock is 500 units, find re-order level, maximum, minimum and average inventory. 7 + 8

9. a) Find an optimal basic feasible solution of the following transportation problem and the optimum transportation schedule and minimum transportation cost. 10

	D_1	D_2	D_3	D_4	Available
O_1	19	30	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
Demand	5	8	7	14	

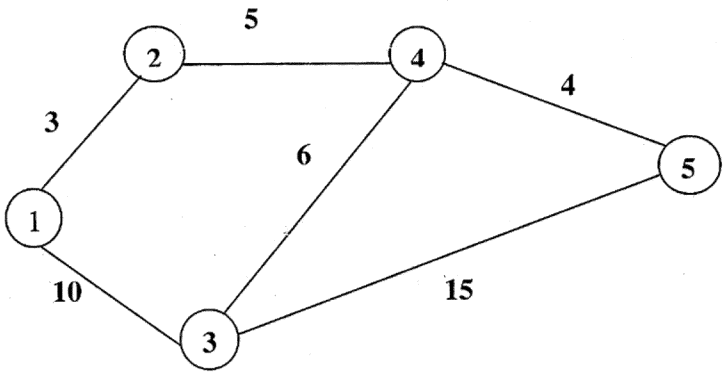
b) Define convex set. Show that $X = \{ x : |x| \leq 2 \}$ is a convex set. 5

10. a) Using Dijkstra's algorithm find the shortest path and the length (or weight) of the shortest path of the following network between node 1 to node 6. 8



OR

Using Floyd's algorithm find the shortest path and the length (or weight) of the shortest path of the following network between node 1 to node 5. 8



b) In the table below list of all the activities which together constitute a small engineering project. The table also displays the necessary immediate predecessors for each activity.

Activity	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
Immediate Predessor	—	*A*	*A*	*A*	*B*	*C, D*	*D*	*B*	*E, F, G*	*G*
Activity duration (in days)	2	3	4	5	6	3	4	7	2	3

You are required to

i) draw the project network

ii) calculate earliest start time and finish time, latest start and finish time of each activity

iii) the critical path

iv) total float for each activity. 7

11. a) Provide the optimum job sequencing involving three machines M_1, M_2, M_3 of which the processing time (in minutes) is as follows and find the total elapsed time and idle time for each machines.

Machine	Jobs					
		J_1	J_2	J_3	J_4	J_5
	A	7	12	11	9	8
	B	8	9	5	6	7
	C	11	13	9	10	14

7

b) A small project consists of seven activities. The details of which are given below :

Activity	Immediate predecessor	Most likely	Optimistic	Pessimistic
A	—	3	1	7
B	A	6	2	14
C	A	3	3	3
D	B, C	10	4	22
E	B	7	3	15
F	D, E	5	2	14
G	D	4	4	4

Draw a network diagram for this project. Find the critical path, the expected project completion time. 8

12. a) Solve the following LPP :

Minimize $Z = 4x_1 + 3x_2$

Subject to $x_1 + 2x_2 \geq 8$

$3x_1 + 2x_2 \geq 12$

and $x_1, x_2 \geq 0$ by Charnes Big M method.

b) Solve the following rectangular game :

		Player B			
		I	II	III	IV
Player A	I	3	5	4	2
	II	5	6	2	4
	III	2	1	4	0
	IV	3	3	5	2

8 + 7